2. Connectives

2.1 Introduction

Goals of this chapter

- Learn our first *metalanguage*, Propositional Logic (**PL**).
- Build a theory of English *connectives*:
  - *and*
  - *or*
  - *not*
  - *if*
- Discuss how these theories bear on theories of reasoning, inference, and some of their limitations and cross-linguistic applications.
- Give precise definitions of core notions in semantics such as *entailment*, *validity*, *consistency*, *contradiction*, *tautology*, and so on.

An example from Benthem et al. 2016.

> In a restaurant, your Father has ordered Fish, your Mother ordered Vegetarian, and you ordered Meat. Out of the kitchen comes some new person carrying the three plates. What will happen?
>
> The waiter asks a first question, say "Who ordered the meat?", and puts that plate. Then he asks a second question "Who has the fish?", and puts that plate. And then, without asking further, he knows he has to put the remaining plate in front of your Mother. What has happened here?

A rough representation of the waiter's reasoning: **meat!** represents the proposition "Mother ordered meat", **fish!** is "Mother ordered fish", and **veg** is "Mother ordered vegetarian".

meat! *or* (**fish!** *or* **veg!**)
not(**meat!**)

not(**fish!**)
∴ **veg!**

This reasoning doesn't necessarily require language, but it's easy to think of a dialogue in which this reasoning is relevant, using our key English connectives.

Kim: 'The lady in blue ordered the fish, meat, **or** vegetarian meal.'
Sandy: 'She did**n't** order the fish, because the man did.'
Alex: 'She did**n't** order the meat, because the boy did.'
Kim: 'So she ordered the vegetarian meal then.

Now we have made our goal clearer: our theory of *or* and *not* should predict this reasoning pattern.

2.2 Connectives and reasoning

2.2.1 Valid inferences

Let's apply this reasoning to a Sudoku puzzle. Let **tr=1** be the proposition 'the top right corner is 1', **tr=2** is 'the top right corner is 2', and **tr=3** is 'the top right corner is 3'.

1	·	·
·	·	2
·	·	·

Write down the reasoning schema which calculates the value for the top right corner. Think about how the schema can be phrased in English sentences.

Valid: An inference is valid if it holds in all contexts in which the premises are true.

What about this reasoning schema?

If you are on a student visa, you are **not** eligible to vote.
You are **not** on a student visa
Therefore, you are eligible to vote.

How does it compare with this one in terms of structure and validity?

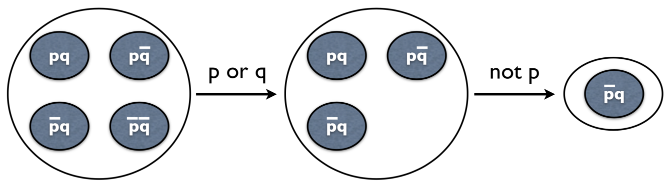
If you are on a student visa, you are **not** eligible to vote.
You are eligible to vote.
Therefore, you are **not** on a student visa.

Logic allows us to predict which inferences are valid and which are invalid, just by looking at their general structure. **PL** predicts the validity of inferences involving 'if', 'not', 'or', and 'and'. This may seem very narrow, but in fact these inferences are *ubiquitous* in language, reasoning, computer science, law, politics, mathematics, and so on.

2.2.2 Reasoning as information update

Logic is a tool to abstract away from the 'content' of inference, and just look at its structure.

Below, p and q are some arbitrary true-or-false statements, while $\bar{p}$ means p is false. Describe how (1) visualizes a reasoning schema.



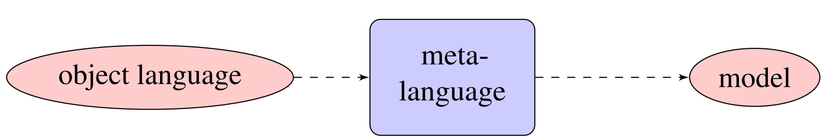
(1)

Consider the case where there are three facts that you are interested in. You wake up, you open your eyes, and you ask yourself three things: "Have I overslept?", "Is it raining?", "Are there traffic jams on the road to work?". To find out about the first question, you have to check your alarm clock, to find out about the second you have to look out of the window, and to find out about the third you have to listen to the traffic info on the radio. We can represent these possible facts with three basic propositions, p, q and r, with p expressing "I have overslept", q expressing "It is raining", and r expressing "There are traffic jams." Suppose you know nothing yet about the truth of your three facts. What is the space of possibilities?

Now you check your alarm clock, and find out that you have not overslept. What happens to your space of possibilities?

2.3 Our first metalanguage: PL

Remember our methodology for doing semantics?



Now we can see this approach in action. We'll use it to build a semantic theory of English connectives. The first step is to define our metalanguage: **PL**.

2.3.1 The syntax of PL

Defining a syntax for **PL** means defining rules for the 'well formed' sentences (just like linguists do for the syntax of natural languages!)

A syntax for PL:

i. The propositions **rain!**, **snow!**, **hot!**, and **cold!** are well formed.
ii. If ϕ and ψ are well formed, then:
   a. $\neg\phi$ is well formed.
   b. $\phi \wedge \psi$ is well formed.
   c. $\phi \vee \psi$ is well formed.
   d. $\phi \rightarrow \psi$ is well formed.
   e. $\phi \leftrightarrow \psi$ is well formed.
iii. Nothing else is well formed.

The following is an abbreviated form of the above.

Let P be a set of propositions, and $p \in P$
$\phi ::= p \mid \neg\phi \mid \phi \vee \phi \mid \phi \wedge \phi \mid \phi \rightarrow \phi \mid \phi \leftrightarrow \phi$

Which are well formed in **PL**? (NB: brackets disambiguate the scope of each operator).

- $\neg$(**rain!** $\vee$ **snow!**) $\rightarrow$ **hot!**
- (($\neg$**hot!** $\vee$ **rain!**) $\rightarrow$ **cold!**
- $\neg\neg\wedge$ **cold!** $\vee$ **rain!**
- **snow!**$\neg$**hot!**

A key point: all we've done is define the syntax of **PL** (i.e., the set of well formed sentences). We have said nothing about what these sentences *mean*.

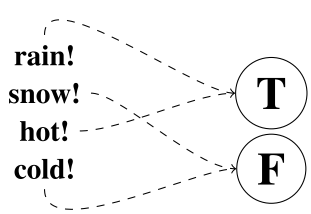
2.3.2 The semantics of PL

Getting at the meaning of a proposition in **PL** is a tricky concept. The following innovative idea comes from the 19th century logicians George Boole and Gottlob Frege.

The semantics of propositions: The meaning of a proposition is **T**(rue) or **F**(alse).

Intuitively, the meaning of **rain!** is **T** if we are in a situation in which it is raining, and **F** if we are in a situation in which is is not raining.

How would you describe the situation below in prose?



Assuming our set of basic propositions only has four members, we can fully specify what a situation is like, so long as each basic proposition is mapped to either **T** or **F**. Draw the situation in which is it cold and snowing, but not hot or raining.

We now have the tools to define a model for interpreting sentences of **PL**.

Models for PL
A model **M** for **PL** is a pair consisting of $\{\mathbf{T},\mathbf{F}\}$ and a function $[\![\ ]\!]^{\mathbf{M}}$

Intuitively, the set $\{\mathbf{T},\mathbf{F}\}$ contains our two truth values. This set is the same in any model for **PL**. The function $[\![\ ]\!]^{\mathbf{M}}$ is used to map propositions to **T** or **F** (just like the dotted arrows above).

$[\![\textbf{rain!}]\!]^{\mathbf{M}} = \mathbf{T}$
$[\![\textbf{snow!}]\!]^{\mathbf{M}} = \mathbf{F}$
$[\![\textbf{hot!}]\!]^{\mathbf{M}} = \mathbf{T}$
$[\![\textbf{cold!}]\!]^{\mathbf{M}} = \mathbf{F}$

How would the function $[\![\ ]\!]^{\mathbf{M'}}$ be specified, where **M'** is the situation in which it is cold and snowing, but not hot or raining?

We can now define a semantics for complex sentences in **PL**, i.e., those containing operators.

The operator ¬ ("bar") flips the truth value of its scope. For any model **M**,

$[\![\phi]\!]^{\mathbf{M}}$	$[\![\neg\phi]\!]^{\mathbf{M}}$
T	**F**
F	**T**

The binary operators have the following effects. *Commit this table to memory!* For any model **M**,

$[\![\phi]\!]^{\mathbf{M}}$	$[\![\psi]\!]^{\mathbf{M}}$	$[\![\phi\wedge\psi]\!]^{\mathbf{M}}$	$[\![\phi\vee\psi]\!]^{\mathbf{M}}$	$[\![\phi\rightarrow\psi]\!]^{\mathbf{M}}$	$[\![\phi\leftrightarrow\psi]\!]^{\mathbf{M}}$
T	**T**	**T**	**T**	**T**	**T**
T	**F**	**F**	**T**	**F**	**F**
F	**T**	**F**	**T**	**T**	**F**
F	**F**	**F**	**F**	**T**	**T**

Handy hints:

- $\phi\wedge\psi$ is true only if both sides are true.
- $\phi\vee\psi$ is false only if both sides are false.
- $\phi\rightarrow\psi$ is true when the left side is false, or the right side is true.
- $\phi\leftrightarrow\psi$ is true when both sides match.

Let's go back to **M** in which it's raining and hot, but not snowing or cold. What are the values of the following propositions?

- $[\![\neg\textbf{snow!}]\!]^{\mathbf{M}} =$
- $[\![\textbf{hot!} \vee \textbf{cold!}]\!]^{\mathbf{M}} =$
- $[\![\neg(\textbf{rain!} \leftrightarrow \textbf{cold!}]\!]^{\mathbf{M}} =$
- $[\![\textbf{cold!} \rightarrow (\textbf{hot!} \wedge \textbf{rain!})]\!]^{\mathbf{M}} =$

FAQs about the **PL** connectives:

- *What's special about these particular operators*?
  Nothing! Nothing prevents us from defining any binary operator we like which assigns different arrangements of four **T** and **F** values. It just so happens that the 5 operators above are most commonly used. Q: How many possible **PL** operators are there?
- *Is it ok to call* $\wedge$ *'and',* $\vee$ *'or', and so on?*
  This is extremely common. You can call the operators what you want, but this can be confusing! These operators are defined **independently** of the English words 'and', 'or', 'not', and 'if'. The operators are expressions of the artificial language **PL**, but 'and', 'or', 'not', and 'if' are expressions of the natural language English. So this abuse of terminology has the potential to be confusing.
- *The definition of* $\vee$ *doesn't match my intuition about 'or'. What gives?*
  As above, $\vee$ and all the operators are defined **independently** of any English words. If we find that 'or' has some properties which $\vee$ doesn't have, that means we need to refine our theory of 'or', rather than change anything about $\vee$.

2.4 PL and making inferences

We now have enough tools to define what it means to make a valid inference:

> **Validity**:
> If $\phi_1, ..., \phi_k$ is a sequence of propositions and ψ is a conclusion, then $\phi_1, ..., \phi_k \models \psi$.
>
> The inference $\phi_1, ..., \phi_k \models \psi$ is valid just in case in any M in which $[\![\phi_1 \wedge, ..., \wedge \phi_k]\!]^{\mathbf{M}} =$ **T**, then $[\![\psi]\!]^{\mathbf{M}}$ is also **T**.

Which of the following inferences are valid?

- **rain! → cold!**
  <u>**rain!**</u>
  cold!
- **rain! → cold!**
  <u>**¬rain!**</u>
  ¬cold!

c. <u>**rain! ∧ cold!**</u>
   cold!

d. <u>**rain! ∨ cold!**</u>
   ¬cold!

Here are some helpful terms to talk about sets of propositions.

a. **Consistent**:
A set of propositions $\phi_1, ..., \phi_k$ is consistent just in case there is some **M** such that $[\![\phi_1 \wedge ... \wedge \phi_k]\!]^{\mathbf{M}} = \mathbf{T}$

b. **Contradiction**:
A set of propositions $\phi_1, ..., \phi_k$ is a contradiction just in case there is no **M** such that $[\![\phi_1 \wedge ... \wedge \phi_k]\!]^{\mathbf{M}} = \mathbf{T}$

c. **Tautology**:
A set of propositions $\phi_1, ..., \phi_k$ is a tautology just in case for all **M**s, $[\![\phi_1 \wedge ... \wedge \phi_k]\!]^{\mathbf{M}} = \mathbf{T}$

Are the following proposition sets contradictions, tautologies, or consistent?

a. **hot!**
rain! $\rightarrow$ **cold!**
$\neg$**cold!** $\wedge$ $\neg$**hot!**

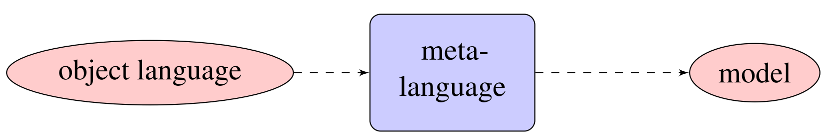
b. **rain!**
snow! $\vee$ $\neg$(**cold!**)
$\neg$**snow!**

c. **rain!** $\vee$ $\neg$**rain!**
hot! $\rightarrow$ **hot!**

Despite its simplicity, **PL** is an immensely powerful and insightful system for understanding inferences. It has broad applications within computer science, engineering, linguistics, philosophy, and any domain in which human reasoning is at issue (e.g., politics, law, advertising, ...)

2.5 PL and linguistics

We can now start to apply our formal system (**PL** and its interpretation in a model) to linguistics! Recall our methodological framework:



We have a fully fleshed out theory of the link between the *metalanguage* and the *model*, via the function $[\![\ ]\!]$. Now we need a link between the *object language* and *metalanguage*.

Translation:
The function $\rightsquigarrow$ maps natural language expressions to metalanguage expressions.

The $\rightsquigarrow$ function is the locus of linguistic theory in semantics. The job of the semanticist is to figure out how to translate expressions natural language expressions into a formal language.

Our first hypothesis: *single-clause, positive polarity sentences translate into basic propositions*.

a. *It's raining* $\rightsquigarrow$ **rain!**
b. *It's snowing* $\rightsquigarrow$ **snow!**
c. *It's hot* $\rightsquigarrow$ **hot!**
d. *It's cold* $\rightsquigarrow$ **cold!**

We can now understand what it means for an English sentence like 'It's raining' to be true (or false) in a model: it's *interpretation* in **PL** denotes **T** (or **F**).

We can also precisely define what it means for a sentence to *entail* another sentence.

> **Entailment**:
> A sentence S ($\rightsquigarrow \phi$) entails a sentence S' ($\rightsquigarrow \psi$) just in case $\phi \models \psi$ is a valid inference.

Let's explore this while we explore the semantics of English connectives.

2.5.1 Negation

We have a preliminary theory of English negation

> **Negation**:
> Where S $\rightsquigarrow \phi$, [*not* S] $\rightsquigarrow \neg\phi$

Of course, [*not* S] doesn't really match the English syntax of negation, let's assume that [*not* S] just stands for the negated version of the sentence S.

Flipping: If $[\![\phi]\!] = \mathbf{T}$, then $[\![\neg\phi]\!] = \mathbf{F}$ and vice versa. Does *not* show this property?

(2) a. Donald Trump is not the President of the USA.
    b. Hillary Clinton is not the President of the USA.

Negation elimination: If $[\![\neg\neg\phi]\!] = \mathbf{T}$, then $[\![\phi]\!] = \mathbf{T}$. Explain why.

(3) a. It's not the case that Donald Trump is not the President of the USA.
    b. Donald Trump isn't not the President of the USA.
    c. It's not the case that Hillary Clinton is not the President of the USA.
    d. Hillary Clinton isn't not the President of the USA.

Negation introduction: If $[\![\phi]\!] = \mathbf{T}$, then $[\![\neg\neg\phi]\!] = \mathbf{T}$. Explain why.

(4) a. Donald Trump is the President of the USA.
    b. Hillary Clinton is the President of the USA.

To evaluate the hypothesis that *not* $\rightsquigarrow \neg$, we examine whether inferences involving $\neg$ correctly predict the entailments of sentences with *not*.

2.5.2 Conjunction

Next, a preliminary theory of English conjunction.

> **Conjunction**:
> Where S $\rightsquigarrow \phi$ and S' $\rightsquigarrow \psi$, [S *and* S'] $\rightsquigarrow \phi \wedge \psi$

Conjunction introduction: If $[\![\phi]\!] = \mathbf{T}$ and $[\![\psi]\!] = \mathbf{T}$, then $[\![\phi \wedge \psi]\!] = \mathbf{T}$.

(5) a. Theresa May is the British prime minister and Jeremy Corbyn is the opposition leader.
b. Theresa May is the British prime minister and Jeremy Corbyn isn't the opposition leader.
c. Boris Johnson is the British prime minister and Theresa May is not the prime minister.

Conjunction elimination: If $[\![\phi \wedge \psi]\!] = \mathbf{T}$, then $[\![\phi]\!] = \mathbf{T}$ and $[\![\psi]\!] = \mathbf{T}$.

(6) What are the entailments of each sentence in (5) using conjunction elimination?

(7) It's not the case that [May is the prime minister and Corbyn isn't not the opposition leader].

NB: If $[\![\phi]\!] = \mathbf{T}$, then we cannot conclude that $[\![\phi \wedge \psi]\!] = \mathbf{T}$ for any ψ

(8) Trump is the president of the US and he will be indicted for breaking campaign finance laws.

2.5.3 Disjunction

Now, a theory of English disjunction.

> **Disjunction**:
> Where S $\rightsquigarrow \phi$ and S' $\rightsquigarrow \psi$, [S *or* S'] $\rightsquigarrow \phi \vee \psi$

Disjunction introduction: If $[\![\phi]\!] = \mathbf{T}$ then $[\![\phi \vee \psi]\!] = \mathbf{T}$, for any ψ. Explain why.

(9) a. Either Xi Jinping is the president of China, or Shinzo Abe is.
b. Either [Xi Jinping is the president of China and Shinzo Abe is the prime minister of Japan] or [Donald Trump is the British prime minister].
c. Either [it's not the case that Xi Jinping isn't the president of China] or [May is the British prime minister].

NB: if $[\![\phi \vee \psi]\!] = \mathbf{T}$ then we cannot conclude that either $[\![\phi]\!] = \mathbf{T}$ or $[\![\psi]\!] = \mathbf{T}$.

(10) Either the UK will leave the EU, or they will hold a second referendum.

2.5.4 Conditionals

Finally, a theory of English conditionals.

> **Conditionals**:
> Where S $\rightsquigarrow \phi$ and S' $\rightsquigarrow \psi$, [*if* S, *then* S'] $\rightsquigarrow \phi \rightarrow \psi$

Modus ponens: The key to making inferences. Where $[\![\phi]\!] = \mathbf{T}$, and $[\![\phi \rightarrow \psi]\!] = \mathbf{T}$, then $[\![\psi]\!] = \mathbf{T}$. This is especially common in contexts of advice, predictions, planning, etc.

(11) a. Interest rates have risen.
b. If interest rates have risen, you should reduce your debt.

(12) a. If Sanders or Biden are selected to win the nomination, Warren will be aggravated.
b. Biden is selected to win the nomination.

Where $[\![\phi]\!] = \mathbf{F}$, and $[\![\phi \rightarrow \psi]\!] = \mathbf{T}$, we cannot draw the inference that $[\![\psi]\!] = \mathbf{F}$.

(13) a. If interest rates have risen, you should reduce your debt.
b. Interest rates have not risen.

(14) a. If you mow the lawn, I'll give you five dollars.
b. You didn't mow the lawn.

NB: sometimes falsifying the antecedent seems to falsify the consequent, as in (14). This is called **Conditional Perfection**. This is *not* predicted by our theory [*if* S, *then* S'] $\rightsquigarrow \phi \rightarrow \psi$. More on this in later weeks.

Modus tollens: Another common principle in inference-making. Where $[\![\phi \rightarrow \psi]\!] = \mathbf{T}$ and $[\![\psi]\!] = \mathbf{F}$, then $[\![\phi]\!] = \mathbf{F}$.

(15) a. If Russia invaded the Ukraine, the EU imposed sanctions on Russia.
b. The EU did not impose sanctions on Russia.

What does *modus tollens* say about the English idiom 'monkey's uncle'?

(16) a. If Brasilia isn't the capital of Brazil, then I'm a monkey's uncle!
b. If this is a real fabergé egg, then I'm the Queen of England!

This is just a *taste* of the hundreds of inference patterns involving $\neg$, $\wedge$, $\vee$, and $\rightarrow$.

Look at the **Wikipedia entry** for a bunch more.

It's an open question whether each of these inference patterns makes good predictions about 'and', 'or', 'not', and 'if'.

2.6 Some limitations of PL

So far, the predictions of associating English connectives with **PL** operators have been relatively good. But there are limitations!

The law of excluded middle: $[\![\phi \vee \neg\phi]\!] = \mathbf{T}$ in any model (i.e., is a tautology).

(17) a. Either Trump is the president or he's not the president.
b. Look, either he's guilty or he's innocent.

The law of non-contradiction: Conversely, $[\![\phi \wedge \neg\phi]\!] = \mathbf{F}$ in any model (i.e., is a contradiction).

(18) a. Clinton is the president and she's not the president.
b. Well, you're right and you're also not right.
c. She's a comedian and yet she's not a comedian.

Both of these principles are especially shaky when it comes to gradable adjectives (e.g., like *happy*, *lucky*, *smart*, etc.)

(19) a. I'm not happy or unhappy – I'm indifferent.
b. He's not tall but he's also not not tall!

Conditionals with false antecedents: Looking at our truth tables, what does the current theory predict about conditionals with false antecedents?

(20) a. If Trump is not the president, then Freddie Mercury is.
b. If Sydney is the capital of Australia, then Melbourne is the capital of Australia.

Strengthening the antecedent: If $[\![\phi \rightarrow \psi]\!] = \mathbf{T}$, then $[\![(\phi \wedge \rho) \rightarrow \psi]\!] = \mathbf{T}$. See if you can work out why. Does this seem like a good prediction for English *if* (example from Stalnaker 1968)?

(21) a. If this match was struck, it would light.
b. If this match had been soaked in water and was struck, it would light.

2.7 Cross-linguistic applications

We've built a theory of English 'and', 'or', 'not', and 'if' (with some gaps). How well does it extend to similar phenomena in other languages?

Several languages (including Arabic, Chinese) have two distinct disjunctions, with the following effects. Examples from Basque (Haspelmath 2007).

(22) a. Te-a **edo** kafe-a nahi duzu?
tea-ART or1 coffee-ART want you.it
Do you want tea or coffee?
b. Te-a **ala** kafe-a nahi duzu?
tea-ART or2 coffee-ART want you.it
Do you want tea or do you want coffee?

The connective *manu* in Warlpiri (Pama-Nyungan; Central Australia) seems to translate as $\vee$ and $\wedge$ in different contexts (from Bowler 2014).

(23) a. Ngapa ka wantimi **manu** warlpa ka wangkami
water AUX fall.NPST *manu* wind AUX speak.NPST
Rain is falling and wind is blowing.
b. Kula-rna yunparnu **manu** wurntija jalangu. Lawa.
NEG-1SG.SUBJ sing.PST *manu* dance.PST today nothing
I didn't sing or dance today. I did nothing.

Horn 1989 partitions four possibilities for encoding connectives using $\wedge$, $\vee$, and $\neg$:

(24) a. $\phi \wedge \psi$: encoded by many (all?) languages (English 'and', French 'et', Tagalog 'at', Samoan "ae', Japanese 'to').
b. $\phi \vee \psi$: encoded by many languages (English 'or', French 'soit...soit' and 'ou', Tagalog 'o', Samoan 'pē', Japanese 'ka').
c. $\neg(\phi \vee \psi)$: encoded by some languages (English 'neither...nor', French 'ni...ni', Dutch 'ook...niet')
d. $\neg(\phi \wedge \psi)$: not encoded by any language (something like 'not both').

An open question: why doesn't $\neg(\phi \wedge \psi)$ ever get encoded in natural language? See Horn 1989, 2007, Katzir and Singh 2013.

2.8 Possible paper topics

- In many languages (including Japanese, Samoan, Malayalam), the particle marking polar questions is the same particle which marks disjunction (e.g., Japanese *ka*). This has received some formal analyses (see e.g., Ciardelli, Groenendijk, and Roelofsen 2018, Uegaki 2018). In other languages, the particle marking embedded questions also marks *implication* (e.g., English *if*, French *si*). This hasn't received as much attention, but is there a potential explanation for this pattern?
- The Warlpiri example in (23) demonstrates a common pattern in the world's languages. Murray 2017 observes a similar pattern in Cheyenne. Löbner 2000 observes something similar in English, in which 'and' gets a $\vee$-like interpretation under negation. Can all these phenomena be given a unified treatment?
- Disjunction in English behaves in unexpected ways with modals (e.g., 'you may have chocolate or vanilla' means something like 'you may have chocoloate and you may have vanilla'). This is called a *free choice inference*. There is a vast literature (see Wright 1968, Kamp 1973, Fox 2007, Franke 2011, Starr 2016 for a taste). But the literature doesn't have a lot of experimental investigation, or much cross-linguistic data.

2.9 Further reading

- Predicate logic is more than 160 years old so there is a vast literature on its properties. Besides the textbook van Benthem et al. 2016, see also Givant and Halmos 2009.
- Logic as applied to linguistics is a smaller literature, but there are still some excellent treatments, especially Winter 2016, Cann, Kempson, and Gregoromichelaki 2009, and Potts 2007 lecture notes.
- The most significant challenges to **PL**-based theories of natural language connectives have targeted the hypothesis that *if* $\rightsquigarrow \rightarrow$. See Stalnaker 1968, Lewis 1973, and Kratzer 1986 for starting points. Nadathur 2013 includes a very readable overview.
- See Haspelmath 2007 for a starting point on the typology of connectives cross-linguistically. See Ramat and Mauri 2011 on grammaticalization paths for connectives.

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