

3. Relationships between words

3.1 Introduction

So far, our semantics using **PL** only gives semantics for (simple and complex) sentences. It says nothing about things smaller than sentences like NPs or VPs.

Here, we make a start by exploring the meanings of words like adjectives, nouns, and verbs, and the relationships between them.

The general strategy is to add more structure to our models. Goals of this handout:

- Move beyond a semantics which only assigns truth values.
- Give more structure to our models
- Propose a semantics for proper names.
- Discuss the meanings of nouns, verbs, and adjectives.
- Explore the basics of set theory.

3.2 Moving inside the sentence

3.2.1 Proper names

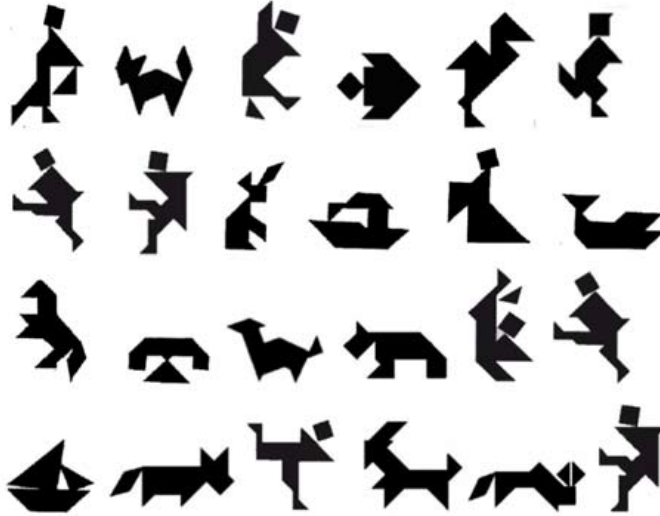
The semantics of proper names (and other kinds of definites) is one of the central questions in the philosophy of language. It is the basis of any notion of *reference/denotation*.

The modern theory dates to Donnellan 1972 and Kripke 1980. This is termed the ‘causal theory of reference’.

“The name is introduced at a formal or informal dubbing. This dubbing is in the presence of the object that will from then on be the bearer of the name. The event is perceived by the dubber and probably others. To perceive something is to be causally affected by it. As a result of this causal action, a witness to the dubbing, if of suitable linguistic sophistication, will gain an ability to use the name to designate the object.” (Devitt and Sterelny 1999:66–67)

A case of ‘dubbing’ in action? Brennan and Clark 1996 observe that interlocutors coordinate on terms for novel objects when they need to repeatedly refer to them.

Participants played a game where they had to both select the same shape from a set of abstract shapes like (1). It resulted in the participants coordinating on simple descriptions and names for each object.



(1)

(2) A: a docksider

B: a what?

A: um

B: is that a kind of dog?

A: no, it's a kind of um leather shoe, kinda preppy pennyloafer

B: okay, okay, got it

‘Dubbing’ can be understood as rooted in principles of rational communication, such as *UID*.

Uniform Information Density: The more predictable an expression, the less structurally complex the expression (see Levy and Jaeger 2007, Jaeger 2010)

A modern case study: Doyle and Frank 2015 argue information flow of this type can be observed in large scale Twitter interactions (e.g., users commenting on the World Series).

- Can we observe in real-time interlocutors converging on a name for a new individual? e.g., ‘the guy in the orange hat sitting in the front row’ → ‘that guy in the orange hat’ → ‘that orange hat guy’ → ‘Mr. Orange Hat’.

How do we incorporate proper names in our theory of semantics? First we have to complicate the model. Currently our models don’t make reference to individuals.

Models (from chapter 2)

A model **M** is a pair, consisting of:

- A set of truth values **{T, F}**
- An interpretation function $\llbracket \rrbracket^{\mathbf{M}}$.

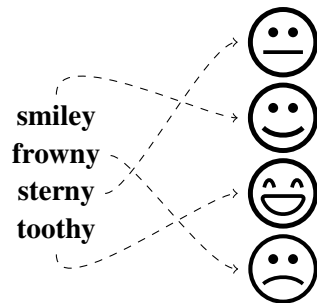
Now we simply add a set of individuals. Intuitively, this is our ‘domain of discourse referents’. The individuals which interlocutors conceivably might refer to (e.g., the 24 shapes in (1)).

Models (updated)

A model \mathbf{M} is a triple, consisting of:

- A set of truth values $\{\mathbf{T}, \mathbf{F}\}$
- A set of individuals U
- An interpretation function $\llbracket \cdot \rrbracket^{\mathbf{M}}$.

Our metalanguage will have to contain terms which refer to individuals. Let’s take $\llbracket \cdot \rrbracket^{\mathbf{M}}$ to be:



What are the values for the following?

- (3)
- a. $\llbracket \text{smiley} \rrbracket^{\mathbf{M}} =$
 - b. $\llbracket \text{toothy} \rrbracket^{\mathbf{M}} =$
 - c. $\llbracket \text{smiley} \rrbracket^{\mathbf{M}} =$
 - d. $\llbracket \neg \text{smiley} \rrbracket^{\mathbf{M}} =$
 - e. $\llbracket \text{smiley} \wedge \text{frowny} \rrbracket^{\mathbf{M}} =$

We can now propose a preliminary theory of *definite NPs*, like proper names, pronouns, and DPs with *the*. As always, English expressions translate into metalanguage expressions.

Semantics of definites:

If an NP is definite, $[\text{NP}] \rightsquigarrow \mathbf{d}$, such that $\llbracket \mathbf{d} \rrbracket$ is an individual in U .

- (4)
- a. *Smiley* \rightsquigarrow **smiley**
 - b. *she* \rightsquigarrow **smiley**
 - c. *the face that’s smiling with no teeth showing* \rightsquigarrow **smiley**
 - d. *Frowny* \rightsquigarrow **frowny**

This turns out to be a pretty horrendous theory of pronouns. Any ideas why?

How does our above theory give insight to the following passage from the *New York Times* (December 29, 2018)?

- (5) After Justice Brett M. Kavanaugh was confirmed to the Supreme Court and in the days before the midterm elections, Mr. Trump told rallygoers in Missouri that “the accuser admitted she never met him, she never saw him, he never touched her, talked to her, he had nothing to do

with her, she made up the story, it was false accusations.” ... The omission of a name and the use of the words “the accuser” may give the misleading impression that Christine Blasey Ford, who testified to Congress that Justice Kavanaugh had sexually assaulted her when they were teenagers, had recanted her account. But in fact, Mr. Trump was referring to another little-known accuser named Judy Munro-Leighton, who recanted her claim of sexual assault.

3.3 Basics of set theory

With a basic theory of definite NPs in tow, we can move on to predicative phrases like adjectives, common nouns, and verbs.

But doing so requires us to get a handle on some basic set theory.

3.3.1 Notation

Curly braces: unordered sets are generally represented using curly braces, with members separated by commas. How would you describe the following set in words?

$$(6) \quad \{ 47, \text{☺}, \diamond \}$$

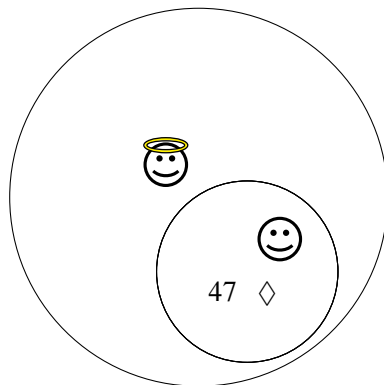
What about this one?

$$(7) \quad \{ \text{☺}, \{ 47, \text{☺}, \diamond \} \}$$

The empty set: the set which doesn't have any members. Why is it called ‘the’ empty set?

$$(8) \quad \emptyset$$

Venn diagrams: Another way to represent sets. Easier to visualize but harder to type. Avoids the implication that an ordering of the members matters.



(9)


Predicate notation: The above notation is great if we can exhaustively list all the members of a set. But this is not often the case. Sometimes we can specify a set using a description.

- (10) a. $\{x \mid x \text{ is a positive integer less than } 7\}$
 b. $\{y \mid y \text{ is a US state on the west coast}\}$

FAQs about predicate notation (from Heim and Kratzer 1998).

- (11) a. What's the difference between $\{y \mid y \text{ is a US state on the west coast}\}$ and $\{x \mid x \text{ is a US state on the west coast}\}$?
 b. What does $\{\text{California} \mid \text{California is a US state on the west coast}\}$ mean?
 c. What does $\{x \mid \text{California is on the east coast}\}$ mean? What about $\{x \mid \text{California is on the west coast}\}$?

Set membership: To say whether or not something is a member of a set, we use \in and \notin .

- (12) a.  $\in \{x \mid x \text{ is smiling}\}$
 b. $47 \notin \{x \mid 0 < x < 100\}$

Let's play True or False.

- (13) a. $\{a\} = \{b\}$
 b. $\{x \mid x = a\} = \{a\}$
 c. $\{x \mid x \text{ is green}\} = \{y \mid y \text{ is green}\}$
 d. $\{x \mid x \text{ likes } a\} = \{y \mid y \text{ likes } b\}$
 e. $\{x \mid x \in A\} = A$
 f. $\{x \mid x \in \{y \mid y \in B\}\} = B$

Recursive definitions: Another way of specifying sets. What set does the following definition describe?

- (14) a. 1 is a member of the set N .
 b. If $d \in N$, then $d + 1 \in N$.
 c. Nothing else is in N .

How could this set be expressed in predicate notation?

Some more FAQs about sets:

- (15) a. What's the difference between $\{\text{😊, 😞}\}$ and $\{\text{😞, 😊}\}$?
 b. What's the difference between $\{\text{😄}\}$ and $\{\text{😄, 😄}\}$?

3.3.2 Relations between sets

Intersection (\cap): The intersection of a set A and set B is the set of things in both A and B .

$$(16) \quad A \cap B := \{x \mid x \in A \ \& \ y \in B\}$$

$$(17) \quad \{\text{😊, 😄, 😞}\} \cap \{\text{😊, 😞, 😇}\} = ?$$

Union (\cup): The union of a set A and set B is the set of things in either A or B .

$$(18) \quad A \cup B := \{x \mid x \in A \ \text{or} \ y \in B\}$$

$$(19) \quad \{\text{😊, 😄, 😞}\} \cup \{\text{😊, 😞, 😇}\} = ?$$

Difference ($-$): The difference between two sets A and B :

$$(20) \quad A - B := \{x \mid x \in A \ \& \ y \notin B\}$$

$$(21) \quad \{ \text{😊, 😊, 😐} \} - \{ \text{😊, 😞, 😇} \} = ?$$

We can take the complement (i.e., the negation) of a set A using $U - A$.

Subset (\subseteq): $A \subseteq B$ defines the subset relationship between sets.

$$(22) \quad A \subseteq B \text{ iff for all } x \in A, \text{ then } x \in B.$$

NB: \subseteq creates a true/false statement, unlike \cap , \cup , and $-$, which take two sets and create a new one. Let's play True or False.

$$(23) \quad \begin{array}{l} \text{a. } \{ \text{😊, 😊} \} \subseteq \{ \text{😊, 😊, 😐} \} \\ \text{b. } \{ \text{😊, 😊, 😇} \} \subseteq \{ \text{😊, 😊, 😐} \} \\ \text{c. } \{ \text{😊, 😊} \} \subseteq \{ \text{😊, 😊} \} \\ \text{d. } \emptyset \subseteq \{ \text{😊, 😊} \} \end{array}$$

Equality ($=$): Equality is defined in terms of subsethood.

$$(24) \quad A = B \text{ iff } A \subseteq B \text{ and } B \subseteq A.$$

Powerset (\wp): The powerset of A is a set containing all subsets of A .

$$(25) \quad \wp(\{ \text{😊, 😞, 😊} \}) = \left\{ \begin{array}{l} \{ \text{😊, 😞, 😊} \}, \\ \{ \text{😊, 😞} \}, \{ \text{😊, 😊} \}, \{ \text{😊, 😞} \}, \{ \text{😊, 😊} \}, \\ \{ \text{😊} \}, \{ \text{😊} \}, \{ \text{😊} \}, \\ \emptyset \end{array} \right\}$$

3.4 Set theory and meaning

3.4.1 Predicates and sets

Now we know set theory, we can start to build a theory of the relationship between lexical items. Our first step, a semantics for predicates, like 'smiles', 'happy', and 'face'.

Semantics of predicates (adjectives, verbs, nouns without complements):

- For any N_{intr} , $[N_{\text{intr}}] \rightsquigarrow \mathbf{pred}$, such that $\llbracket \mathbf{pred} \rrbracket \subseteq U$.
- For any Adj_{intr} , $[Adj_{\text{intr}}] \rightsquigarrow \mathbf{pred}$, such that $\llbracket \mathbf{pred} \rrbracket \subseteq U$.
- For any V_{intr} , $[V_{\text{intr}}] \rightsquigarrow \mathbf{pred}$, such that $\llbracket \mathbf{pred} \rrbracket \subseteq U$.

Let's update a few things. First, let's update our models to accommodate predicates.

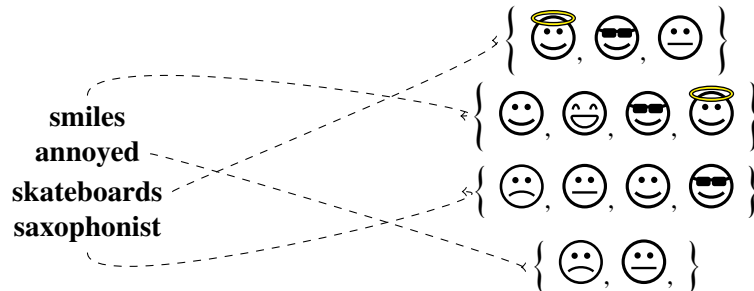
Models (updated)

A model \mathbf{M} is a triple, consisting of:

- i. A set of truth values $\{\mathbf{T}, \mathbf{F}\}$

- ii. A set of individuals U
- iii. A set of sets of individuals $A \subseteq \wp(U)$
- iv. An interpretation function $\llbracket \cdot \rrbracket^M$.

Step (iii) allows us to map predicates like **smiles**, **happy**, **face** to denotations. Let's specify $\llbracket \cdot \rrbracket^M$ as:



Let's play true or false.

- (26)
- a. $\llbracket \text{smiles} \rrbracket^M \cap \llbracket \text{saxophonist} \rrbracket^M \neq \emptyset$
 - b. $\llbracket \text{annoyed} \rrbracket^M \subseteq \llbracket \text{skateboards} \rrbracket^M$
 - c. $(\llbracket \text{annoyed} \rrbracket^M - \llbracket \text{skateboards} \rrbracket^M) \subseteq \llbracket \text{smiles} \rrbracket^M$
 - d. $\llbracket \text{annoyed} \rrbracket^M \in \wp(\llbracket \text{saxophonist} \rrbracket^M)$

Let's play 'Don't Do What Donny Don't Does'.

- (27)
- a. According to our theory, and $\rightsquigarrow \wedge$, $\text{smiles} \rightsquigarrow \text{smiles}$, and $\text{skateboards} \rightsquigarrow \text{skateboards}$. Therefore $\llbracket [\text{smiles}] \text{ and } [\text{skateboards}] \rrbracket \rightsquigarrow \text{smiles} \wedge \text{skateboards}$. What went wrong?
 - b. In our M above, All the annoyed individuals are saxophonists, therefore $\llbracket \text{annoyed} \rrbracket^M \in \llbracket \text{saxophonist} \rrbracket^M$. What went wrong?

Summary of our three 'types' of expressions

Expression	Category	ML Translation	Denotation
<i>it's raining</i>	S	rain!	T
<i>Smiley</i>	DP	smiley	😊
<i>is happy</i>	VP	happy	{😊, 😎}

Our theory that predicates map to sets via $\llbracket \cdot \rrbracket$ gives us a powerful way of understanding lexical relations like synonymy, hyponymy and so on.

3.4.2 Lexical relationships

Understanding the relationships between sub-sentential is crucial for making inferences. What are the practical applications of detecting entailments in the following pairs?

- (29)
- a. Every (firm) [polled] {saw costs grow more than expected}.
 - b. Every (big company) [in the poll] {reported cost increases}.

from MacCartney and Manning 2009

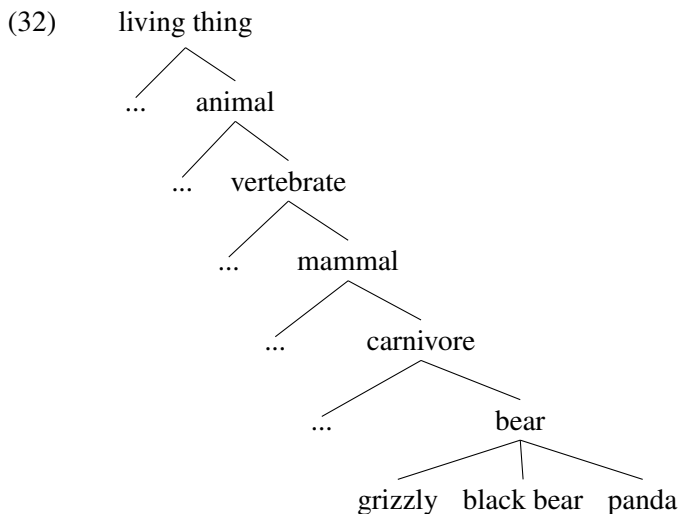
- (30) a. The [restaurants] (often) {have a sort of pan-Asian flair} and there are [many sushi bars].
 b. The [kitchens] (rarely) {have any sort of pan-Asian flair} and there are [numerous Japanese food bars].
 from Riemer 2010

Hyponymy

Hyponymy is the lexical relation described in English by the phrase *kind/type/sort of*, e.g.,

- (31) a. *blues – jazz – music*
 b. *ski-parka – parka – jacket*
 c. *commando – soldier – member of armed forces*
 d. *martini – cocktail – drink*
 e. *paperback – book*

A significant application for hyponymies, *taxonomy*: classifications of natural kinds, artifacts, etc.



We can define *hyponymy* formally using our current tools. How?

(33)

What can we say about folk taxonomies, e.g., *tomato/cucumber* vs. *fruit/vegetable*?

Synonymy

At a basic level, **synonymy** is the lexical relation of identity. Two expressions are synonymous if they have the same meaning.

- (34) a. *likely – probable*
 b. *couch – sofa*
 c. *pupil – student*
 d. *baggage – luggage*

At this level of complexity, we can characterize synonymy. How?

(35)

How do we detect synonymy? Able to be substituted in all contexts?

- (36)
- vomit – barf*
 - urinate – go powder my nose*
 - nei[i]ther – nei[ai]ther*
 - Bombay – Mumbai*

What's the relationship between synonymy and hyponymy?

Are the following synonymous? Hyponomous? What does our theory predict?

- (37)
- teaches LING623 – male*
 - animals with hearts – animals with kidneys*
 - spy – alleged spy*
 - natto – delicious*

Non-subsective relations

MacCartney and Manning 2009 look at semantic relationships besides subsethood.

	Relation	Definition	Example?
	negation	$A \cap B = \emptyset \ \& \ A \cup B \neq U$	
(38)	alternation	$A \cap B = \emptyset \ \& \ A \cup B = U$	
	cover	$A \cap B \neq \emptyset \ \& \ A \cup B = U$	
	overlap	$A \cap B \neq \emptyset \ \& \ A \cup B \neq U$	

Can we draw these relations graphically?

How would you characterize the following?

- (39)
- happy – unhappy*
 - hungry – hippo*
 - mouse – elephant*
 - hot – cold*
 - skiing – sleeping*
 - skiing – eating*
 - French – German*
 - not French – not German*

3.5 Relations

Sets are denotations for *one-place predicates*, i.e., expressions which are true of one individual.

But some expressions are true of two or more individuals, like *love*, *tease*, *wash*, *give*, *show*, *friend-of*, *mother-of* and so on. We need to use the notion of *tuples* or *ordered sets* to understand this.

3.5.1 Ordered sets

A *tuple* (or *ordered set*) is a finite sequence of elements. The following tuple has 😊 as its first member, and the number 18 as its second.¹

$$(40) \quad \langle \text{😊}, 18 \rangle$$

Ordering matters in tuples, such that (40) is different from (41).

$$(41) \quad \langle 18, \text{😊} \rangle$$

Which pairs of the following sets are equivalent?

- (42) a. $\langle \text{😊}, \text{😐} \rangle$
 b. $\{ \text{😊}, \text{😐} \}$
 c. $\langle \text{😐}, \text{😊} \rangle$
 d. $\{ \text{😐}, \text{😊} \}$
 e. $\{ \text{😊} \}$
 f. $\langle \text{😊} \rangle$

3.5.2 Relation

A relation is a set of tuples. A 2-place relation is a set of ordered pairs, a 3 place relation is a set of ordered triples, and so on. An example of a 2-place relation:

$$(43) \quad \{ \langle \text{😊}, \text{😊} \rangle, \langle \text{😇}, \text{😎} \rangle, \langle \text{😐}, \text{😊} \rangle \}$$

An example of a 2-place relation with predicate notation.

$$(44) \quad \{ \langle x, y \rangle \mid x \text{ is a parent of } y \}$$

A Cartesian product of two sets A and B is $A \times B$

$$(45) \quad A \times B = \{ \langle x, y \rangle \mid x \in A \ \& \ y \in B \}$$

$$(46) \quad \{a, b\} \times \{1, 2\} = \left\{ \begin{array}{l} \langle a, 1 \rangle, \langle a, 2 \rangle \\ \langle b, 1 \rangle, \langle b, 2 \rangle \end{array} \right\}$$

3.5.3 Relations in semantics

Now we know relations, we can use them to model the meanings of ≥ 2 -place relations, like *love*, *friend*, *own*, etc.

Semantics of 2-place relations (adjectives, verbs, nouns with complements):

- For any N_{tr} , $[N_{tr}] \rightsquigarrow \mathbf{rel}$, such that $[[\mathbf{rel}]] \subseteq U \times U$.

¹Benthem et al. 2016 uses $()$ instead of $\langle \rangle$, this is just a notational choice.

- For any Adj_{tr} , $[\text{Adj}_{\text{tr}}] \rightsquigarrow \mathbf{rel}$, such that $\llbracket \mathbf{rel} \rrbracket \subseteq U \times U$.
- For any V_{tr} , $[\text{V}_{\text{tr}}] \rightsquigarrow \mathbf{rel}$, such that $\llbracket \mathbf{rel} \rrbracket \subseteq U \times U$.

Obviously, we need to generalize this for 3-place, 4-place relations and so on.

To incorporate this, we need to update the model.

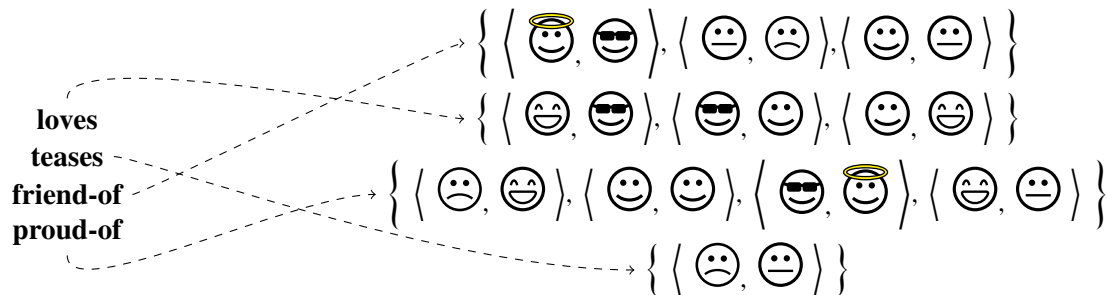
Models (updated)

A model \mathbf{M} consists of:

- A set of truth values $\{\mathbf{T}, \mathbf{F}\}$
- A set of individuals U
- For any $n > 1$, a set of relations $A \subseteq \wp(U_1 \times \dots \times U_n)$
- An interpretation function $\llbracket \cdot \rrbracket^{\mathbf{M}}$.

Although n is defined to be any number, we really only use 2- and 3-place relations in natural language for all intents and purposes.

Here are some sample denotations for some 2-place relations.



And what about a three place relation:



Let's play true or false.

- (47)
- $\langle \llbracket \text{smiley} \rrbracket^{\mathbf{M}}, \llbracket \text{toothy} \rrbracket^{\mathbf{M}} \rangle \in \llbracket \text{loves} \rrbracket^{\mathbf{M}}$
 - $\llbracket \text{teases} \rrbracket^{\mathbf{M}} \cap \llbracket \text{friend-of} \rrbracket^{\mathbf{M}} \neq \emptyset$
 - $\llbracket \text{loves} \rrbracket^{\mathbf{M}} \subseteq \llbracket \text{proud-of} \rrbracket^{\mathbf{M}}$

Updating our 'types' of expressions

Expression	Category	ML Translation	Denotation
<i>it's raining</i>	S	rain!	T
<i>Smiley</i>	DP	smiley	
(48) <i>skateboards</i>	V_{intr}	skateboards	$\{ \langle \text{smiley}, \text{toothy} \rangle \}$
<i>teases</i>	V_{tr}	teases	$\{ \langle \text{neutral}, \text{neutral} \rangle, \langle \text{toothy}, \text{smiley} \rangle \}$
<i>introduces</i>	V_{ditr}	introduces	$\{ \langle \text{neutral}, \text{neutral}, \text{smiley} \rangle, \langle \text{toothy}, \text{smiley}, \text{smiley} \rangle \}$

3.5.4 Relations between relations

Our 1-place relations (e.g., *happy*, *skateboards*) are sets. Our > 1 -place relations (e.g., *loves*, *friend-of*) are *also* sets (of tuples).

Because relations are sets, our theories of hyponymy and synonymy can be maintained without any changes.

(49) **Hyponymy:**

'A' $\rightsquigarrow \alpha$ is a hyponym for 'B' $\rightsquigarrow \beta$ iff in all models M , $\llbracket \alpha \rrbracket^M \subseteq \llbracket \beta \rrbracket^M$.

Some relational cases of hyponymy.

- (50) a. *touch* – *rub*
 b. *perceive* – *see* – *watch*
 c. *discuss* – *criticize*

Here's an example illustrating hyponymy via set theory.

- (51) a. $\llbracket \text{see} \rrbracket^M = \{ \langle \text{😊}, \text{😎} \rangle, \langle \text{😐}, \text{😞} \rangle, \langle \text{😊}, \text{😞} \rangle \}$
 b. $\llbracket \text{watch} \rrbracket^M = \{ \langle \text{😊}, \text{😎} \rangle, \langle \text{😐}, \text{😞} \rangle \}$

Synonymy works basically the same way.

(52) **Synonymy:**

'A' $\rightsquigarrow \alpha$ is a synonym for 'B' $\rightsquigarrow \beta$ iff in all models M , $\llbracket \alpha \rrbracket^M = \llbracket \beta \rrbracket^M$.

An example of synonymy between sets:

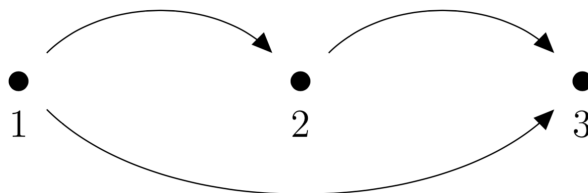
- (53) a. $\llbracket \text{forbid} \rrbracket^M = \{ \langle \text{😊}, \text{😎} \rangle, \langle \text{😐}, \text{😞} \rangle \}$
 b. $\llbracket \text{prohibit} \rrbracket^M = \{ \langle \text{😊}, \text{😎} \rangle, \langle \text{😐}, \text{😞} \rangle \}$

3.5.5 Types of relations

Relations can have various formal properties. These can help us understand entailment patterns of natural language expressions.

Transitivity

- (54) A relation R is **transitive** iff it holds for all x, y, z that if $\langle x, y \rangle \in R$ and $\langle y, z \rangle \in R$, then also $\langle x, z \rangle \in R$.



(55)

(56) Which of the following relations are transitive?

- $\{(1, 2), \langle 2, 3 \rangle, \langle 3, 4 \rangle\}$
- $\{(1, 2), \langle 2, 3 \rangle, \langle 3, 4 \rangle, \langle 1, 3 \rangle, \langle 2, 4 \rangle\}$
- $\{(1, 2), \langle 2, 3 \rangle, \langle 3, 4 \rangle, \langle 1, 3 \rangle, \langle 2, 4 \rangle, \langle 1, 4 \rangle\}$
- $\{(1, 2), \langle 2, 1 \rangle\}$
- $\{(1, 1), \langle 2, 2 \rangle\}$

Some (ML translations of) English expressions which are plausibly transitive relations.

- (57)
- in-front-of, behind, to-the-left-of, to-the-right-of**
 - taller-than, shorter-than, wider-than** and so on.
 - any others?
 - What are some non-transitive relations?

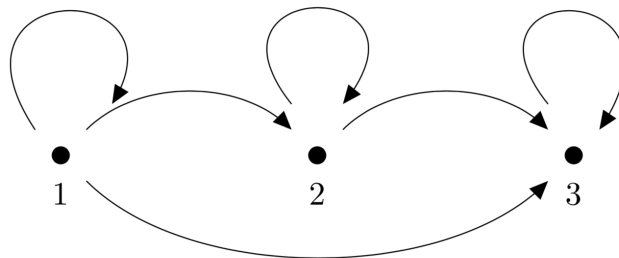
If the above relations are transitive, we correctly predict the following entailments:

- (58)
- Smiley is taller than Frowny.
Frowny is taller than Sterny.
Therefore, Smiley is taller than Sterny
 - Toothy is behind Cool.
Cool is behind Frowny.
Therefore, Toothy is behind Frowny.

Reflexivity

(59) A relation R is **reflexive** iff it holds for all x , $\langle x, x \rangle \in R$.

The following is a representation of a transitive and reflexive relation.



(60)

List the set of pairs in the relation (60).

Some (ML translations of) English expressions which are plausibly reflexive relations.

- (61)
- is-identical-to, is-the-same-thing-as**
 - as-tall-as, as-short-as**, and so on
 - any others?
 - What are some non-reflexive relations?

Some plausible inferences based on reflexive relations.

- (62) a. Smiley is the same person as Smiley.
b. Smiley is as tall as Smiley.

Some other relational properties

- (63) **Irreflexivity** (the opposite of reflexivity)
A relation R is **irreflexive** iff it holds for all x , $\langle x, x \rangle \notin R$.
- (64) **Symmetry**
A relation R is **symmetrical** iff it holds for all x, y , if $\langle x, y \rangle \in R$, then $\langle y, x \rangle \in R$.
- (65) **Antisymmetry**
A relation R is **antisymmetrical** iff it holds for all x, y , if $\langle x, y \rangle \in R$ and $\langle y, x \rangle \in R$, then $x = y$.
- (66) **Asymmetry**
A relation R is **asymmetrical** iff it holds for all x, y , if $\langle x, y \rangle \in R$, then $\langle y, x \rangle \notin R$.

Look at the **Wikipedia** for more types of relations.

- (67) How would you classify the following English expressions by the types of relations they denote.
- on top of*
 - friend of*
 - mother of*
 - sibling of*
 - attached to*
 - hug*

3.6 Possible paper topics

- The Donnellan-Kripke theory of proper names (employing ‘dubbing’ or ‘baptism’) is meant to explain how humans derive proper names for real world objects. Can we derive a more structured model of how humans choose proper names for objects which predicts a preference for simpler names for more commonly referred to objects. Do Schelling games (of the sort described in Brennan and Clark 1996, Potts 2008) give insight?
- We talked about the relations *hyponymy*, *synonymy* etc. We left aside discussion of *antonymy* (e.g., *big* vs. *small*, *hero* vs. *coward*), which is much harder to characterize formally (see Riemer 2010: §5.1.1). It seems to involve gradability/scalarity to some extent (see upcoming Handout 6, Rett 2007, 2015). Is there a principled approach to characterizing antonymy and what are its applications?
- The notion of *mereology* (e.g., *finger* vs. *hand*, and *steering wheel* vs. *bus*) could also use some investigation (see Riemer 2010: §5.1.2). There is extensive work on semantic applications of mereology (see Champollion and Krifka 2016 for an overview). But the relationship is mainly focused on relationships like *drop of water* to *water*. What about more specific kinds of part-whole relationships, like, e.g., body parts?
- Relevant to taxonomies is the notion of *prototypes*, e.g., ‘a robin is a prototypical bird’ but ‘an emu is not a prototypical bird’. Can our formal notion of taxonomies incorporate a notion

of prototypes/typicality? See Tiel 2013, 2014 for an application of typicality for a different function.

- It's actually quite difficult to think of pairs of words which are truly synonymous inasmuch as they are used in the same contexts (e.g., consider differing contexts for *vomit* vs. *spew*). Is there a way to model why lexicons tend to prefer pairs of words becoming dissimilar? Maybe there's insight from the literature on phonetic distinctiveness (e.g., Flemming 2004).
- Metaphorical extensions of expressions is a fascinating domain of linguistics, which has received relatively little attention in formal linguistics (though see Kao, Bergen, and Goodman 2014, Cohn-Gordon and Qing 2019 for some recent approaches). Do these theories (or modifications of them) give insight into how lexical items are related metaphorically (synchronically and diachronically).

3.7 Further reading

- As always, Partee, Meulen, and Wall 1990 is full of detail and exercises to help understand set theory, relations, and their applications in linguistics.
- Kracht 2003 is a more advanced text on similar topics.
- MacCartney and Manning 2009 is a hugely insightful short paper about natural language technology using the kinds of set theoretic notions discussed above, which has applications to theoretical linguistics, psycholinguistics, our understanding of the lexicon and so on.
- See Cruse 1986 and Murphy 2002 for higher level, more philosophical notions of relations between lexical items and the notion of concepts.

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