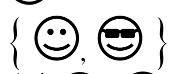
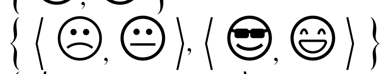
4. Introduction to Quantifiers

4.1 Introduction

We now have an understanding of the semantics of full sentences, definites, and $\geq$ 1-place relations.

	Expression	Category	ML Translation	Denotation
	it's raining	S	**rain!**	**T**
	Smiley	DP	**smiley**	
(1)	*skateboards*	V_{intr}	**skateboards**	
	teases	V_{tr}	**teases**	
	introduces	V_{ditr}	**introduces**	

But so far, we haven't seen how to put these expressions together. That's the focus of this handout.

To do this, we start to look at **quantifiers**. As with connectives, quantifiers are crucial to understanding human reasoning and inference.

The approach we take follows from the 19th century philosophy Gottlob Frege. It requires us to adopt a new metalanguage to replace **PL**. We will now work with **FOL** (*First Order Logic*).

4.2 Syllogistic reasoning

4.2.1 Introduction

Aristotle (384 BC – 322 BC) proposed a system of *syllogistic* reasoning in order to understand the semantics of statements involve quantifiers like *all* and *some*.

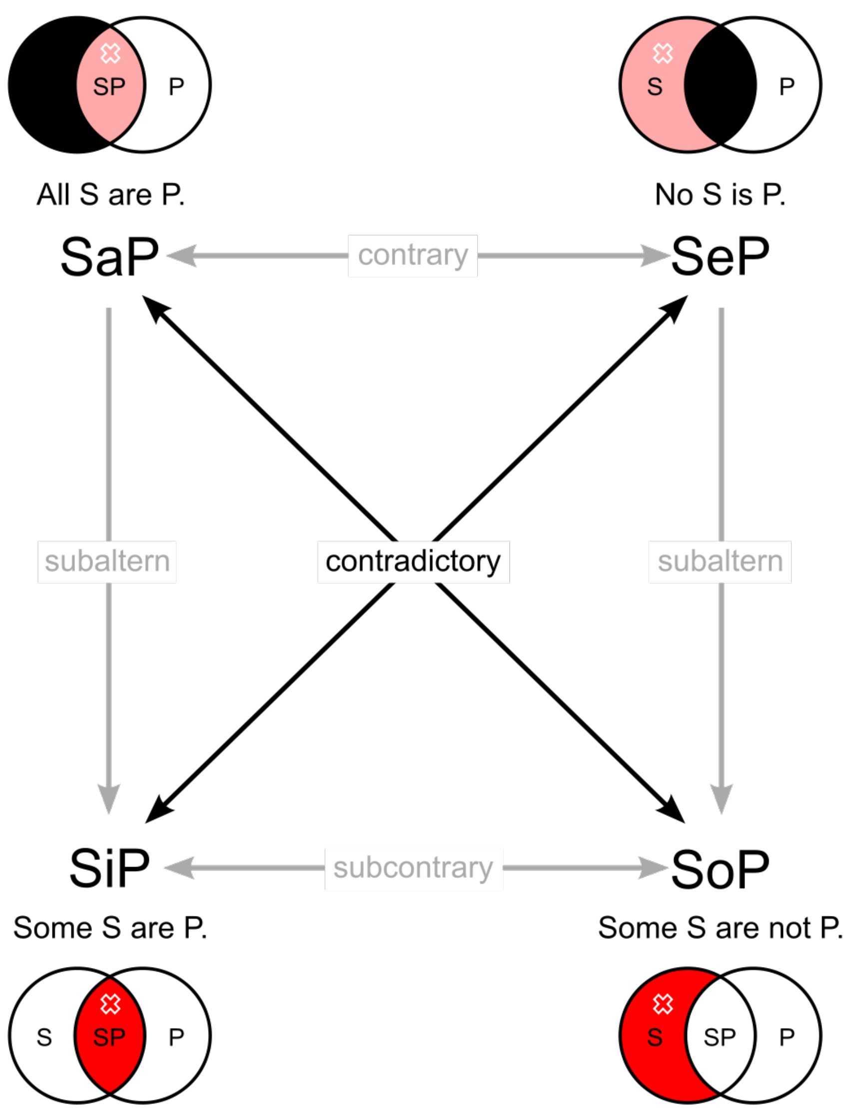
Syllogisms are *reasoning schemas* which lay out a skeleton of a valid inference. For example:

(2) All A are B
All B are C
All A are C.

We can use these to license specific inferences, e.g.:

(3) All citizens are eligible for jury duty
All individuals eligible for jury duty are eligible to work.
All citizens are eligible to work.

Syllogisms focus on four quantifiers: *all*, *some*, *no*, and *not all* (or *some ... not*). These comprise the **Square of Opposition** (image from **Wikipedia**):



(4)

Note that Aristotle assumed his quantifiers had **Existential Import**:

(5) **Existential Import**:
A statement of *No/All* A *are* B implies that there is at least one A.

Modern logicians don't assume Existential Import, but assuming it renders *all* and *no* closer to their English equivalents.

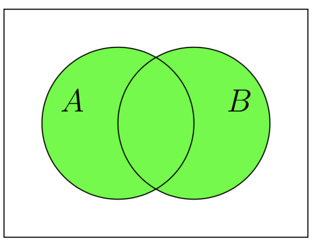
4.2.2 Syllogisms, Sets, and Venn Diagrams

We can use the set theory we learned in the previous handout to understand how syllogisms work.

We can treat predicates like A and B in syllogisms as sets and perform our basic set theoretic operations on them. Here's an example:

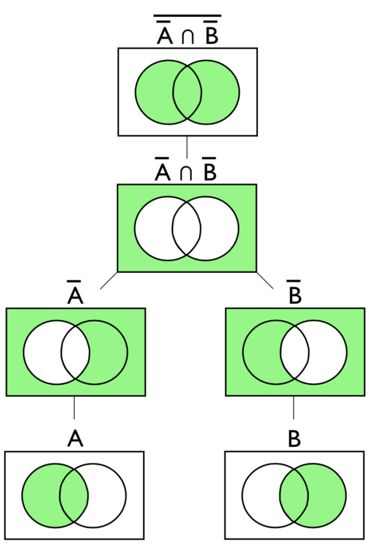
(6) Jerry is either a citizen or an organ donor.
Jerry is not both a non-citizen and a non-organ donor.

The premise can be represented as below, where green means "Jerry is in this region". This region is derived by taking the *union* of the two sets.



(7)

Now we can derive the conclusion componentially, proving its equivalence to the premise.



(8)

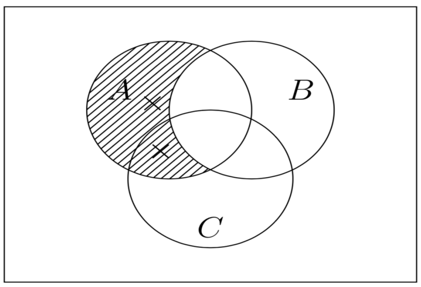
Let's try an example on our own. Can we prove the following equivalence via the same method?

(9) Jerry is both a citizen and an organ donor.
Jerry is not either a non-citizen or a non-organ donor.

Let's move to quantificational statements, starting with the positive quantifiers:

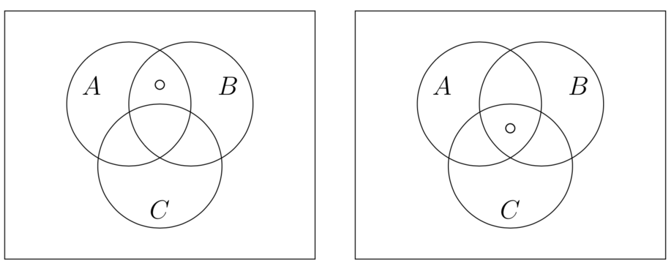
(10) a. **All** A **are** B: The region of A outside of B is empty.
b. **Some** A **are** B: The region where A and B overlap is not empty.

Graphically, the *All* statement is represented by crossing out the empty region.



(11)

The *Some* statement is represented by placing an individual in the non-empty region. Why are there two pictures here?



(12)

Let's prove this equivalence:

(13) All citizens are organ donors.
It's not the case that some citizens are non-organ donors.

What about the negative quantifiers.

(14) a. **No** A **is** B: The region where A and B overlap is empty.
b. **Not all** A **are** B: The region of A outside of B is non-empty.

Observations:

i. Each negative quantifier is a negation of a positive quantifier.
ii. All four quantifiers are defined in terms of (non-)emptiness of sub-regions of A.

Let's represent the negative quantifiers graphically (on the board).

Now we can prove some more complex syllogisms.

(15) All Republican senators supported repealing Obamacare.
No democratic socialist supported repealing Obamacare.
No Republican senator is a democratic socialist.

Step by step:

i. Draw an empty domain of discourse with three properties.
ii. Cross out forbidden regions.
iii. Populate non-empty regions.
iv. Flip (non-)empty regions for any negations.
v. Check conclusion.

One more, valid or invalid?:

(16) No members of the Australian Nationals supported same-sex marriage
Some supporters of same-sex marriage were conservative.
Not all conservatives are members of the Australian Nationals.

4.3 Updating our metalanguage: individuals and predicates

Now we are in a position to start theorizing about the semantics of sentences comprised of predicates.

Our working metalanguage **PL** is not powerful enough to do this.

PL assigns truth values (**T** and **F**) to sentences. Thus we can use **PL** to understand the relationship between "It's raining", and "It's raining and it's not snowing", for example.

But we have no way of detecting the relationships between sentences like the following:

(17) a. Angela is a lawyer.
b. Angela is a poet.
c. Some poets are lawyers.
d. No lawyers are who are poets are also gardeners.

The relationships between these sentences are due to expressions *below* the level of the sentence.

We need a new, updated metalanguage to handle the meanings of sub-sentential expressions like 'some', 'lawyer', 'Angela', and so on. We will use **FOL** (*First Order Logic*).

4.3.1 The Syntax of individuals and predicates

FOL is an extension of **PL**, so all the features of **PL** still remain. A reminder:

A syntax for PL:

i. Any proposition p in a set of basic propositions Prop (e.g., **rain!**, **snow!**, **hot!**, and **cold!**) is well formed.
ii. If ϕ and ψ are well formed, then:
  a. $\neg\phi$ is well formed.
  b. $\phi \wedge \psi$ is well formed.
  c. $\phi \vee \psi$ is well formed.
  d. $\phi \rightarrow \psi$ is well formed.
  e. $\phi \leftrightarrow \psi$ is well formed.
iii. Nothing else is well formed.

The following is an abbreviated form of the above.

> Let Prop be a set of propositions, and $p \in \Phi$
> $\phi ::= p \mid \neg\phi \mid \phi \vee \phi \mid \phi \wedge \phi \mid \phi \to \phi \mid \phi \leftrightarrow \phi$

Let's add to it:

> **A syntax for FOL** (not complete):
>
> i. Any proposition p in a set of basic propositions Prop (e.g., **rain!**, **snow!**, **hot!**, and **cold!**) is well formed.
> ii. If ϕ and ψ are well formed, then, $\neg\phi$, $\phi \wedge \psi$, $\phi \vee \psi$, and $\phi \to \psi$ are well formed.
> iii. Take Pred to be a set of $n \geq 1$-place relations (e.g., **loves**, **lawyer**, **republican**, **proud-of**), and Inds to be a set of individual terms (e.g., **smiley**, **frowny**, **toothy**).
>   a. If R_1 is a 1-place relation (a property) and a is an individual, $R_1(a)$ is well formed.
>   b. If R_2 is a 2-place relation and a_1, a_2 are individuals, $R_2(a_1, a_2)$ is well formed.
>   c. If R_3 is a 3-place relation and a_1, a_2, a_3 are individuals, $R_3(a_1, a_2, a_3)$ is well formed.
> iv. Nothing else is well formed.

Can we simplify/generalize (iii) above?

Let's use the abbreviated notation:

> Let Prop be a set of propositions, and $p \in \Phi$
> Let Pred be a set of $n \geq 1$-place relations, and $R_n \in \text{Pred}$
> Let Inds be a set of individual terms, and $a_1, ..., a_n \in \text{Inds}$
> $\phi ::= p \mid R_n(a_1, ..., a_n) \mid \neg\phi \mid \phi \vee \phi \mid \phi \wedge \phi \mid \phi \to \phi \mid \phi \leftrightarrow \phi$

Which of the following are well-formed formulas in **FOL**?

(18)
a. $\neg(\textbf{rain!} \vee \textbf{snow!}) \to \textbf{hot!}$
b. $\neg\neg \wedge \textbf{loves}(\textbf{smiley}, \textbf{frowny})$
c. $\textbf{lawyer}(\textbf{smiley}, \textbf{frowny}, \textbf{toothy})$
d. $\textbf{skateboards}(\textbf{angel}) \wedge \neg\textbf{proud-of}(\textbf{smiley}, \textbf{cool})$

Let's stop here, and figure out the meanings of **FOL** sentences so far.

4.3.2 The semantics of individuals and predicates

We need a way of interpreting **FOL** sentences like **skateboards(angel)**. Other than that, everything is the same as **PL**.

Let's remind ourselves of what model looks like from the previous handout.

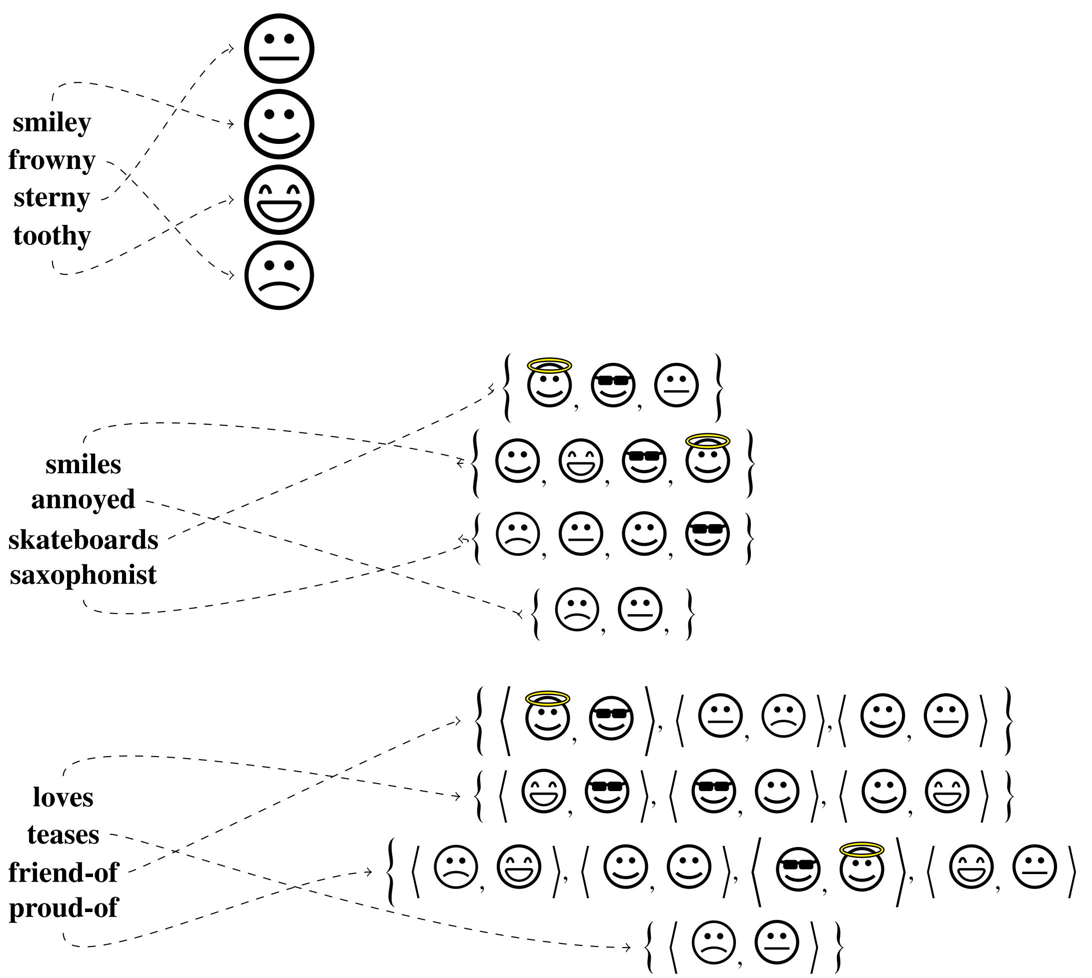
Models (from Handout 3)
A model $\mathbf{M}$ consists of:

i. A set of truth values $\{\mathbf{T},\mathbf{F}\}$
ii. A set of individuals U
iii. For any $n > 1$, a set of relations $A \subseteq \wp(U_1 \times ... \times U_n)$
iv. An interpretation function $[\![\ ]\!]^{\mathbf{M}}$.

We also already have interpretation rules for individual terms and relations, relative to a model.

(19) a. For any individual term $a \in \textsf{Inds}$, $[\![a]\!]^{\mathbf{M}} \in U$
b. For any relation $R_n \in \textsf{Pred}$, $[\![R_n]\!]^{\mathbf{M}} \subseteq \wp(U_1 \times ... \times U_n)$

We've already seen examples of this. Here's an example of a possible specification of $[\![\ ]\!]^{\mathbf{M}}$:



Now we need an interpretation rule for predicates-with-arguments!

(20) a. For any formula $R_1(a) = \mathbf{T}$ iff $[\![a]\!]^{\mathbf{M}} \in [\![R_1]\!]^{\mathbf{M}}$
b. For any formula $R_2(a,b) = \mathbf{T}$ iff $\langle [\![a]\!]^{\mathbf{M}}, [\![b]\!]^{\mathbf{M}} \rangle \in [\![R_2]\!]^{\mathbf{M}}$
c. For any formula $R_3(a,b,c) = \mathbf{T}$ iff $\langle [\![a]\!]^{\mathbf{M}}, [\![b]\!]^{\mathbf{M}}, [\![c]\!]^{\mathbf{M}} \rangle \in [\![R_3]\!]^{\mathbf{M}}$
d. and so on...

How do we simplify/generalize this?

(21) a. For any formula $R_1(a) = \mathbf{T}$ iff $[\![a]\!]^{\mathbf{M}} \in [\![R_1]\!]^{\mathbf{M}}$
b. For any formula $R_n(a_1, ..., a_n) = \mathbf{T}$ iff $\langle [\![a_1]\!]^{\mathbf{M}}, ..., [\![a_n]\!]^{\mathbf{M}} \rangle \in [\![R_n]\!]^{\mathbf{M}}$

Which of the following formulas of **FOL** are true relative to **M** above?

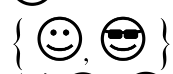
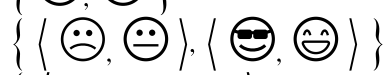
(22) a. $\mathbf{smiles(cool)}$
b. $\neg\mathbf{smiles(sterny)}$
c. $\mathbf{friend\text{-}of(angel, cool)} \vee \neg\mathbf{annoyed(frowny)}$
d. $\mathbf{saxophonist(smiley)} \rightarrow \neg\mathbf{teases(frowny, sterny)}$

Come up with two true and two false sentences of **FOL** relative to **M**.

4.4 Mapping English to FOL: individual and predicates

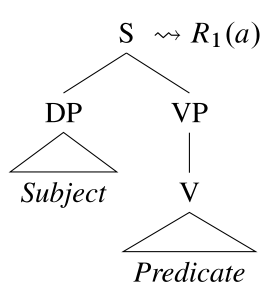
In the last handout we had some rules mapping English to predicates/individual terms. To recap:

(23)

Expression	Category	ML Translation	Denotation
it's raining	S	**rain!**	**T**
Smiley	DP	**smiley**	
skateboards	V_{intr}	**skateboards**	
teases	V_{tr}	**teases**	
introduces	V_{ditr}	**introduces**	

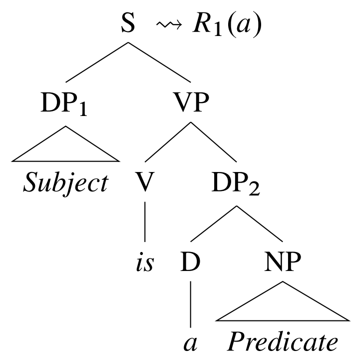
Now we need rules getting us from *definite* DPs (e.g., proper names, definites, pronouns) and VPs to sentences.

(24) **Rule 1: Subject-intransitive VPs**
If DP $\rightsquigarrow a$ and VP $\rightsquigarrow R_1$, then

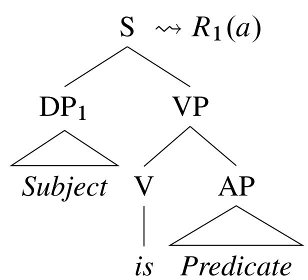


The next rule handles adjectival and nominal predicate, assuming English *is* and *a* are vacuous.

(25) **Rule 1a: Nominal predicates**
If $\mathrm{DP}_i \rightsquigarrow a$ and $\mathrm{NP} \rightsquigarrow R_1$, then



(26) **Rule 1b: Adjectival predicates**
If $\mathrm{DP}_1 \rightsquigarrow a$ and $\mathrm{AP} \rightsquigarrow R_1$, then



What are the **FOL** translations of the following?

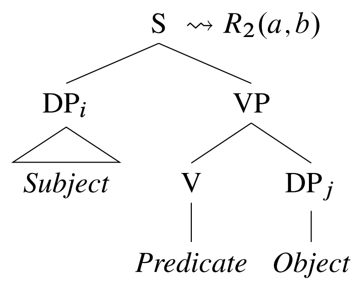
(27) a. Smiley is happy.
b. Two is a prime number.
c. If Frowny is annoyed, then Toothy is not skateboarding.
d. Frowny is not annoyed and two is not a prime number.

Looking at the model **M** two pages back, are the following English sentences true or false?

(28) a. Smiley is a saxophonist.
b. If Cool is annoyed then Angel is not smiling.
c. Either Sterny is annoyed or Angel skateboards.
d. It's not the case that Angel is not smiling.

Next we need a semantics for English transitives.

(29) **Rule 2: Transitive predicates**
If $\mathrm{DP}_i \rightsquigarrow a$, $\mathrm{DP}_j \rightsquigarrow b$ and $\mathrm{V} \rightsquigarrow R_2$, then



What are the **FOL** translations of the following?

(30) a. Kim loves Sandy.
b. If either Smiley is skateboarding or Frowny is annoyed, then Frowny teases Smiley.
c. 2 is not greater than -3.
d. Neither Sterny loves Cool nor Cool teases Sterny.

Based on the model **M**, are the following sentences true or false?

(31) a. Frowny teases Sterny.
b. Cool loves Smiley and Toothy doesn't love Cool.
c. If Frowny is annoyed then he teases Cool.
d. Either Angel doesn't skateboard or Toothy loves Smiley.

You'll extend this to ditransitives, transitive nominals/adjectives in your homework.

4.5 Quantification and FOL

4.5.1 The Syntax of Quantification in FOL

We understand how to combine individual terms with predicates, we can move to quantification.

We need to finish defining the syntax of **FOL**. Now we'll include quantification.

The most crucial addition is a set of variables. These are just like individual terms (e.g., **smiley**, **toothy**), except their value is undetermined.

Like individual constants, variables can be arguments of predicates.

A syntax for FOL (complete):
Take Prop to be a set of basic propositions (e.g., **rain!**, **snow!**, etc.)
Take Pred to be a set of $n \geq 1$-place relations (e.g., **teases**, **skateboards**, etc.)
Take Inds to be a set of individual constants (e.g., **smiley**, **toothy**, etc.)
Take Var to be a set of individual variables (e.g., x, y, etc.)

i. Any proposition $p \in \mathsf{Prop}$ is well formed.
ii. If ϕ and ψ are well formed, then, $\neg\phi$, $\phi \wedge \psi$, $\phi \vee \psi$, and $\phi \rightarrow \psi$ are well formed.
iii. If $R_n \in \mathsf{Pred}$ and $a_1, ..., a_n \in \mathsf{Inds} \cup \mathsf{Var}$, then $R_n(a_1, ..., a_n)$ is well formed.
iv. If $x \in \mathsf{Var}$ and ϕ is well formed, then $\forall x[\phi]$ and $\exists x[\phi]$ are well formed.
v. Nothing else is well formed.

Which of the following are well formed formulas in **FOL**?

(32)
a. $\mathbf{smiles}(x)$
b. $\forall x[\mathbf{smiles}(x)]$
c. $\forall x[\mathbf{smiles}(\mathbf{cool})]$
d. $\exists x[\mathbf{smiles}(x) \wedge \mathbf{lawyer}(y)]$
e. $\exists \mathbf{smiley}[\mathbf{skateboards}(\mathbf{smiley})]$
f. $\exists[\mathbf{skateboards}(\mathbf{smiley})]$
g. $\neg\exists x[\mathbf{skateboards}(x) \wedge \forall y[\mathbf{skateboards}(y) \rightarrow \forall z[\mathbf{happy}(z) \rightarrow \mathbf{introduce}(x,y,z)]]]$

4.5.2 The semantics of the universal quantifier

The **FOL** sentence $\forall x[\phi]$ is 'universally quantified'.

$\llbracket\phi\rrbracket^{\mathbf{M}}_{x:=d}$ means take ϕ and replace every instance of $\llbracket x\rrbracket_{x:=d}$ with d.[1]

(33) **Semantics of universal quantification**
$\llbracket\forall x[\phi]\rrbracket^{\mathbf{M}} = \mathbf{T}$ iff $U \subseteq \{d \mid \llbracket\phi\rrbracket^{\mathbf{M}}_{x:=d}\}$

Working through an example:

(34) $\llbracket\forall x[\mathbf{smiles}(x)]\rrbracket = \mathbf{T}$
$= U \subseteq \{d \mid \llbracket\mathbf{smiles}(x)\rrbracket_{x:=d}\}$
$= U \subseteq \{d \mid \llbracket x\rrbracket_{x:=d} \in \llbracket\mathbf{smiles}\rrbracket\}$
$= U \subseteq \{d \mid d \in \llbracket\mathbf{smiles}\rrbracket\}$
$= U \subseteq \llbracket\mathbf{smiles}\rrbracket$

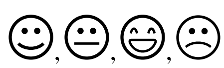
Slightly more complicated:

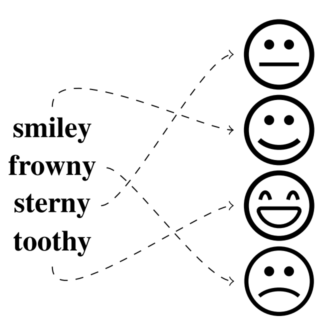
(35) $\llbracket\forall x[\mathbf{introduces}(\mathbf{smiley}, x, \mathbf{cool})]\rrbracket = \mathbf{T}$
$= U \subseteq \{d \mid \llbracket\mathbf{introduces}(\mathbf{smiley}, x, \mathbf{cool})\rrbracket_{x:=d}\}$
$= U \subseteq \{d \mid \langle\llbracket\mathbf{smiley}\rrbracket, \llbracket x\rrbracket_{x:=d}, \llbracket\mathbf{cool}\rrbracket\rangle \in \llbracket\mathbf{introduces}\rrbracket\}$
$= U \subseteq \{d \mid \langle$  $, d,$  $\rangle \in \llbracket\mathbf{introduces}\rrbracket\}$

There are other equivalent ways of conceptualizing universal quantification.

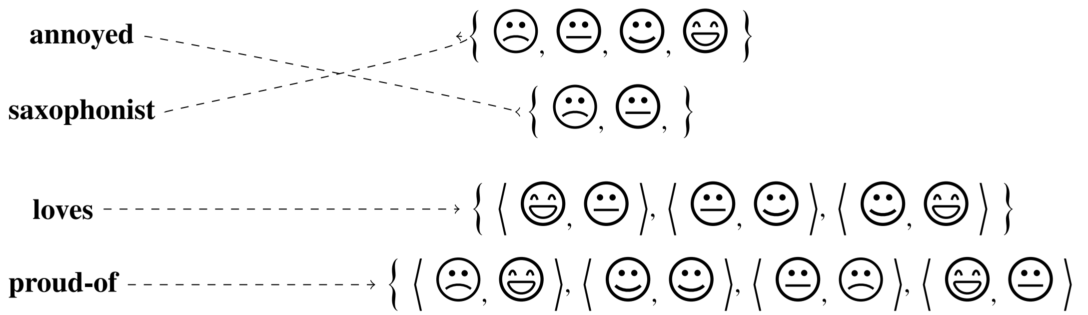
(36) $\llbracket\forall x[\mathbf{smiles}(x)]\rrbracket = \mathbf{T}$ iff for every $d \in U$, $d \in \llbracket\mathbf{smiles}\rrbracket$

(37) Where $U = \{a, b, c\}$, $\llbracket\forall x[\mathbf{smiles}(x)]\rrbracket = \mathbf{T}$ iff $a \in \llbracket\mathbf{smiles}\rrbracket$ & $b \in \llbracket\mathbf{smiles}\rrbracket$ & $c \in \llbracket\mathbf{smiles}\rrbracket$

Here's a new **M**. $U = \Big\{$  $\Big\}$ and $\llbracket\ \rrbracket^{\mathbf{M}}$:



[1] We need an extra stipulation to ensure that x is not 're-quantified' within ϕ.



Are the following **T** or **F** relative to **M** above?

(38)
a. $\forall x[\textbf{saxophonist}(x)]$
b. $\forall x[\textbf{saxophonist(smiley)}]$
c. $\forall x[\textbf{saxophonist}(x) \vee \textbf{annoyed}(x)]$
d. $\forall x[\textbf{loves}(x, \textbf{smiley})]$
e. $\forall x[\textbf{saxophonist}(x) \rightarrow \textbf{annoyed}(x)]$

(e) in (38) is a very common combination of operators. Let's explore it a bit:

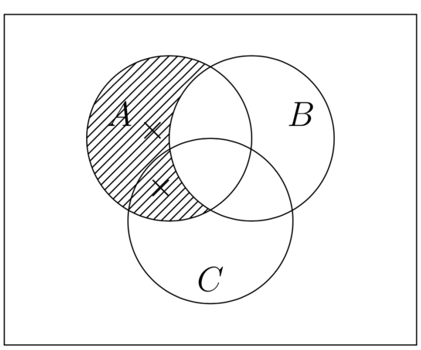
(39)
$$\begin{aligned}
&[\![\forall x[\textbf{saxophonist}(x) \rightarrow \textbf{annoyed}(x)]]\!]^{\mathbf{M}} = \mathbf{T} \text{ iff}\\
&U \subseteq \{d \mid [\![\textbf{saxophonist}(x) \rightarrow \textbf{annoyed}(x)]\!]^{\mathbf{M}}_{x:=d}\}\\
&U \subseteq \{d \mid [\![\textbf{saxophonist}(x)]\!]^{\mathbf{M}}_{x:=d} = \mathbf{F} \text{ or } [\![\textbf{annoyed}(x)]\!]^{\mathbf{M}}_{x:=d} = \mathbf{T}\}\\
&U \subseteq \{d \mid d \notin [\![\textbf{saxophonist}]\!]^{\mathbf{M}} \text{ or } d \in [\![\textbf{annoyed}]\!]^{\mathbf{M}}\}
\end{aligned}$$

This states that it's impossible for an individual to be a **saxophonist** while not also being **annoyed**.

A much more intuitive, and totally equivalent, characterization:

(40)
$$[\![\forall x[\textbf{saxophonist}(x) \rightarrow \textbf{annoyed}(x)]]\!]^{\mathbf{M}} = \mathbf{T} \text{ iff } [\![\textbf{saxophonist}]\!] \subseteq [\![\textbf{annoyed}]\!]$$

So the ($\forall \ldots \rightarrow$) combo reduces to subsethood.



(41)

4.5.3 The semantics of the existential quantifier

The **FOL** sentence $\exists x[\phi]$ is 'existentially quantified'. Here's the semantics:

(42) **Semantics of existential quantification**
$[\![\exists x[\phi]]\!]^{\mathbf{M}} = \mathbf{T}$ iff $\{d \mid [\![\phi]\!]^{\mathbf{M}}_{x:=d}\} \neq \emptyset$

Working through an example.

(43) $[\![\exists x[\textbf{smiles}(x)]]\!] = \mathbf{T}$
$= \{d \mid [\![\textbf{smiles}(x)]\!]^{\mathbf{M}}_{x:=d}\} \neq \emptyset$
$= \{d \mid d \in [\![\textbf{smiles}]\!]^{\mathbf{M}}\} \neq \emptyset$
$= [\![\textbf{smiles}]\!]^{\mathbf{M}} \neq \emptyset$

A more complicated example:

(44) $[\![\exists x[\textbf{introduces}(\textbf{smiley}, x, \textbf{cool})]]\!] = \mathbf{T}$
$= \{d \mid [\![\textbf{introduces}(\textbf{smiley}, x, \textbf{cool})]\!]_{x:=d}\} \neq \emptyset$
$= \{d \mid \langle [\![\textbf{smiley}]\!], [\![x]\!]_{x:=d}, [\![\textbf{cool}]\!] \rangle \in [\![\textbf{introduces}]\!]\} \neq \emptyset$
$= \{d \mid \langle$ ☺ $, d,$ 😎 $\rangle \in [\![\textbf{introduces}]\!]\} \neq \emptyset$

Thus $\exists x[\phi]$ means that ϕ is true of at least one individual. Here are some equivalent ways of conceptualizing existential quantification.

(45) $[\![\exists x[\textbf{smiles}(x)]]\!] = \mathbf{T}$ iff for at least one $d \in U$, $d \in [\![\textbf{smiles}]\!]$

(46) Where $U = \{a, b, c\}$, $[\![\exists x[\textbf{smiles}(x)]]\!] = \mathbf{T}$ iff $a \in [\![\textbf{smiles}]\!]$ or $b \in [\![\textbf{smiles}]\!]$ or $c \in [\![\textbf{smiles}]\!]$

Let's go back to the model **M** above. Are the following statements **T** or **F** relative to **M**?

(47) a. $\exists x[\textbf{saxophonist}(x)]$
b. $\exists x[\neg\textbf{saxophonist}(x) \vee \textbf{annoyed}(x)]$
c. $\exists x[\textbf{loves}(x, \textbf{smiley})]$
d. $\exists x[\textbf{saxophonist}(x) \wedge \textbf{proud-of}(x, \textbf{toothy})]$

(d) in (38) is also a very common combination of operators. Let's explore:

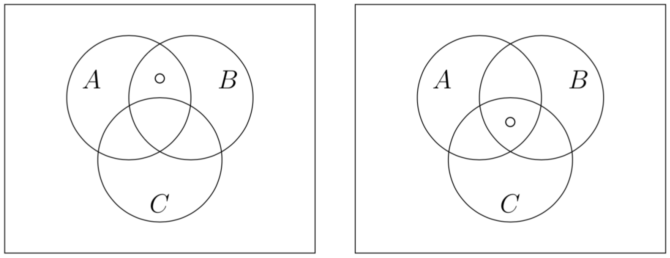
(48) $[\![\exists x[\textbf{saxophonist}(x) \wedge \textbf{annoyed}(x)]]\!] = \mathbf{T}$
$\{d \mid [\![\textbf{saxophonist}(x) \wedge \textbf{annoyed}(x)]\!]_{x:=d}\} \neq \emptyset$
$\{d \mid d \in [\![\textbf{saxophonist}]\!]\} \cap \{d \mid d \in [\![\textbf{annoyed}]\!]\} \neq \emptyset$
$[\![\textbf{saxophonist}]\!] \cap [\![\textbf{annoyed}]\!] \neq \emptyset$

This states that the overlapping region of **saxophonist** and **annoyed** is not empty.

Again, we have a simpler (equivalent) shortcut.

(49) $[\![\exists x[\textbf{saxophonist}(x) \wedge \textbf{annoyed}(x)]]\!] = \mathbf{T}$ iff $[\![\textbf{saxophonist}]\!] \cap [\![\textbf{annoyed}]\!] \neq \emptyset$

So the $(\exists ... \wedge)$ combo reduces to non-empty overlap.

(50) 

4.6 Mapping English to FOL: Quantifiers

Remember that we already have a rule for interpreting English DPs.

> **Semantics of definites**:
> If an NP is definite, [NP] $\rightsquigarrow$ **d**, such that $[\![\mathbf{d}]\!]$ is an individual in U.

Can quantifiers be interpreted via the same rule? No! Why not? Arguments from Heim and Kratzer 1998:§6.1.

Argument 1: Assuming $[\![\textbf{Swedish student}]\!] \subseteq [\![\textbf{student}]\!]$, individuals support upward inferences. Ask yourself why.

(51) Smiley is a Swedish student.
Smiley is a student.

However, not all quantifiers have the same property.

(52) No one is a Swedish student.
$\not\models$ No one is a student.

Argument 2: Assuming $[\![\textbf{dog}]\!] \cap [\![\textbf{cat}]\!] = \emptyset$, individuals lead to contradictions.

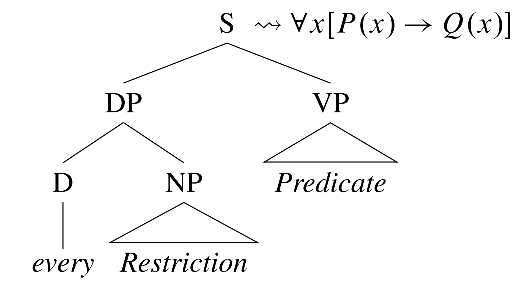
(53) Smiley is a dog.
Smiley is a cat.
(*contradiction*)

Again, this isn't true of all quantifiers.

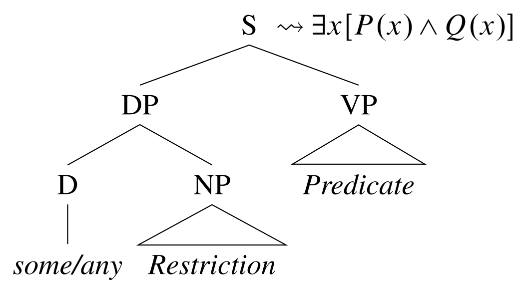
(54) Something is a dog.
Something is a cat.
(*consistent*)

Obviously, we need new rules for quantificational DPs. Assume VP is intransitive or has a definite object throughout.

(55) **Rule 4a: Quantificational subject: *every***
If NP $\rightsquigarrow P$ and VP $\rightsquigarrow Q$, then



(56) **Rule 4b: Quantificational subject: *some/any***
If NP $\rightsquigarrow P$ and VP $\rightsquigarrow Q$, then



(57)
a. Every saxophonist skateboards.
b. Every saxophonist loves Smiley.
c. Some student doesn't smoke.
d. Either every saxophonist smokes or some lawyer doesn't tease Frowny.

Looking at the most recent model **M**, are the following English sentences true or false?

(58)
a. Every saxophonist is annoyed.
b. Some annoyed individuals are saxophonists.
c. Some individual who is proud of Smiley loves Toothy.
d. Every saxophonist is proud of Frowny.

Now we can use our theory of quantifiers to understand syllogistic reasoning. Is the following valid, why? why not?

(59) Some Mediterranean countries are in the EU.
It's not the case that any EU countries border the Indian Ocean
―――――――――――――――――――――――――――――――
It's not the case that any Mediterranean countries border the Indian Ocean

4.7 Possible paper topics

- There are various proposals for the semantics of exceptives, such as *unless* and *except*, including Fintel 1992, Leslie 2008, Nadathur 2014, and Nadathur and Lassiter 2014. Some proposals suggest *unless* has a different meaning depending on whether it is embedded beneath a positive or negative quantifier. Are there cross-linguistic insights on this question? What open questions are there in the literature?

- Coppock and Beaver 2015 observe that *Anna didn't eat the only carrot cake (at the bake sale)* has a very different interpretation to *Anna didn't bake the only carrot cake (at the bake sale)*. They analyze the difference due to an ambiguity in semantics of *the only*. But how much of a role does the verb play? See also the response in Bumford 2017.
- Bumford 2015 observes cases in English which our basic notion of *every* doesn't obviously extend to, such as *Every year Mary wrote a more interesting book* or *Every house on this street has more and more Christmas decorations.* What are the conditions that give rise to this type of reading? How do notions of time or space play a role?
- Certain expressions have been analyzed as being interpreted variously as $\forall$ and $\exists$ depending on their environment. This is famously described in relation to reciprocal expressions like *each other* by Dalrymple et al. 1998. Some more recent work focuses on plurals with *the* like Löbner 2000 and Kriz 2015. What are the cross-linguistic applications of this? For example, Bittner and Hale 1995 suggest that Warlpiri has a word ambiguous between an *every*- and *many*-type meaning (see also Bowler 2014 on Warlpiri). Bar-Lev and Margulis 2014 suggest something similar for a quantifier in Modern Hebrew. What are the empirical and theoretical foundations of these kinds of expressions? What predictions do these competing theories make and are they borne out?

4.8 Further reading

- $\sim$3200 years on, syllogistic reasoning remains as relevant as ever. See Horn 1989 for a very influential, classic text in lingustics about negation including its role in syllogisms. See also Khemlani and Johnson-Laird 2012 and Tessler and Goodman 2014 for some modern approaches to syllogisms in language and the psychology behind them.
- There are many explorations of the mapping of logic (including **FOL**) to natural language: Partee, Meulen, and Wall 1990, Gamut 1991a, 1991b, Carpenter 1997, and Winter 2016 are excellent.
- Probably the most influential exploration of how syntax maps to semantics is Heim and Kratzer 1998, itself applying Montague's theories (e.g., Montague 1973) to a GB-style syntax.
- An excellent recent theory of the syntax-semantics interface is outlined in Barker 2015, 2016 and Barker and Shan 2014.

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