5. More on Quantification

5.1 Introduction

Let's review our current metalanguage **FOL**:

A syntax for FOL (complete):
Take Prop to be a set of basic propositions (e.g., **rain!**, **snow!**, etc.)
Take Pred to be a set of $n \geq 1$-place relations (e.g., **teases**, **skateboards**, etc.)
Take Inds to be a set of individual constants (e.g., **smiley**, **toothy**, etc.)
Take Var to be a set of individual variables (e.g., x, y, etc.)

i. Any proposition $p \in \mathsf{Prop}$ is well formed.
ii. If ϕ and ψ are well formed, then, $\neg\phi$, $\phi \wedge \psi$, $\phi \vee \psi$, and $\phi \rightarrow \psi$ are well formed.
iii. If $R_n \in \mathsf{Pred}$ and $a_1, ..., a_n \in \mathsf{Inds} \cup \mathsf{Var}$, then $R_n(a_1, ..., a_n)$ is well formed.
iv. If $x \in \mathsf{Var}$ and ϕ is well formed, then $\forall x[\phi]$ and $\exists x[\phi]$ are well formed.
v. Nothing else is well formed.

The most important additions in Handout 4 were the $\forall$ and $\exists$ operators, which allow us to handle quantification. We proposed semantics for *some*, *every*, *any*, *no*, and *not every*.

But there are a ton more quantifiers in English which don't yet have obvious analyses:

(1) a. Most of the students smoke.
    b. Exactly four of the students smoke.
    c. At least two but no more than ten students smoke.
    d. Few/many students smoke.

Goals for this handout:
- Explain why **FOL** isn't sufficient to analyze every English quantificational expression.
- Incorporate a new notion, **Generalized Quantifier**, into our metalanguage.

5.2 Limitations of FOL

Currently we have no obvious way of handling this:

(2) Most students smoke.

Intuitively, what does this sentence mean to you?

Maybe the most obvious route is to add a new quantifier to **FOL**, let's call it M_o, such that $[\![M_o x[\phi]]\!] = \mathbf{T}$ just in case ϕ is true for over half of U.

Do any of the following sentences represent a reading of (2)? For each formula, describe a scenario in which it differs in truth from (2).

(3) a. $M_o x[\mathbf{student}(x) \wedge \mathbf{smoke}(x)]$

b. $M_o x[\mathbf{student}(x) \vee \mathbf{smoke}(x)]$

c. $M_o x[\mathbf{student}(x) \rightarrow \mathbf{smoke}(x)]$

Therefore, **FOL** is inadequate in analyzing *most*, and any proportional quantifier (e.g., *(more/less than) half, two out of three*) has the same issue.

We also find that analyzing numerical expressions in **FOL** results in confoundingly complicated formulas.

(4) Two students smoke $\rightsquigarrow \exists x[\exists y[\mathbf{student}(x) \wedge \mathbf{student}(y) \wedge x \neq y \wedge \mathbf{smoke}(x) \wedge \mathbf{smoke}(y)]]$

Any ideas about how to write these ones in **FOL**?

(5) a. Either eight or eleven students smoke.
b. An infinite number of students smoke.
c. An even number of students smoke.

The notion of *Generalized Quantifier* is designed to overcome these limitations.

5.3 Generalized Quantifiers

Let's summarize what kind of expressions we have in our metalanguage:

(6)

Expression	Category	ML Translation	Denotation
it's raining	S	**rain!**	**T**
Smiley	DP	**smiley**	☺
skateboards	V_{intr}	**skateboards**	{ ☺, 😎 }
teases	V_{tr}	**teases**	{ ⟨ ☹, 😐 ⟩, ⟨ 😎, 😄 ⟩ }
introduces	V_{ditr}	**introduces**	{ ⟨ ☹, 😐, 😇 ⟩, ⟨ 😎, 😄, ☺ ⟩ }

Now we're going to add another type, the quantifier.

Starting with an analogy, we already have n-place relations over individuals, e.g.

(7) a. **skateboards**: a 1-place relation over individuals (or a property).
b. **teases**: a 2-place relation over individuals.
c. **introduces**: a 3-place relation over individuals.
d. and so on...

A quantifier: an n-place relation over properties. For example a quantifier Q might have the denotation $[\![Q]\!] = \{[\![\textbf{smile}]\!], [\![\textbf{happy}]\!], [\![\textbf{cat}]\!], [\![\textbf{smoke}]\!], [\![\textbf{skateboards}]\!]\}$.

It's for this reason that GQs are called "higher order properties": they are properties of properties!

5.3.1 Some 1-place quantifiers

Let's start with our bread and butter: *everything*, *something/anything*, *nothing*, and *not everything*.

First an additional rule for well-formed formulas in our updated metalanguage.

(8) **New syntax rule:**
If Q is a 1-place quantifier, and P is a property (i.e., a 1-place relation) then $\mathsf{Q}(P)$ is a well formed formula.

With the addition of (9), we're already updating the metalanguage. Our new metalanguage is now called "two sorted type theory" (or $\mathbf{TY_2}$ for short). Why? We'll see later on.[1]

Of course we need a rule for interpreting $\mathsf{Q}(P)$.

(9) **New semantics rule:**
If $\mathsf{Q}(P)$ is a well formed formula, $[\![\mathsf{Q}(P)]\!] = \mathbf{T}$ iff $[\![P]\!] \in [\![\mathsf{Q}]\!]$.

Basically $\mathsf{Q}(P)$ just says the property P is one of the properties in Q.

Universal quantifiers

The 1-place quantifier **everything** holds of any property P, just in case P is true of every individual in the relevant discourse universe U. Let's sketch that on the board.

(10) $[\![\textbf{everything}]\!] = \{P \mid U \subseteq P\}$

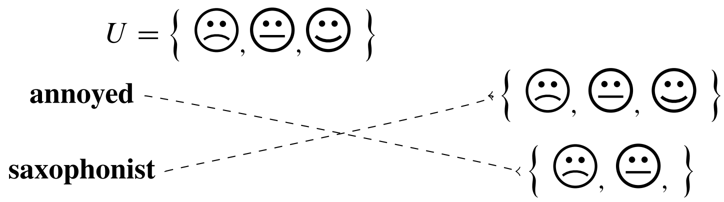
The 1-place quantifier **not.everything** is just the negation. This means that it is the set of all properties P not in $[\![\textbf{everything}]\!]$.

(11) $[\![\textbf{not.everything}]\!] = \{P \mid U \not\subseteq P\}$

Question: what is $[\![\textbf{everything}]\!] \cap [\![\textbf{not.everything}]\!]$? What about $[\![\textbf{everything}]\!] \cup [\![\textbf{not.everything}]\!]$?

Here's a mini-model.

[1] The name comes from Gallin 1975.



(12) a. $[\![\textbf{everything}]\!]^M =$
b. $[\![\textbf{not.everything}]\!]^M =$

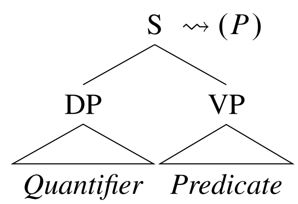
(13) a. $[\![\textbf{everything(saxophonist)}]\!]^M =$
b. $[\![\textbf{not.everything(annoyed)}]\!]^M =$

Now we need rules mapping English to expressions in $\mathbf{TY}_2$.

(14) a. *everything* $\rightsquigarrow$ **everything**
b. *not everything* $\rightsquigarrow$ **not.everything**

A major bonus of this new approach: we only need one rule for interpreting quantificational subjects. With **FOL** in the last handout, we needed a different one for each quantifier.

(15) **Rule Q: Quantificational subjects**
If DP $\rightsquigarrow$ Q, a quantifier, and VP $\rightsquigarrow$ P, then

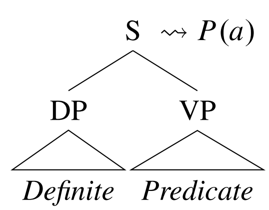


Are the following English sentences true or false relative to M?

(16) a. *Not everything is a saxophonist.*
b. *Everything is annoyed.*

Rule Q stands beside the rule for definite subjects, now called **Rule Def**, which is the same as before:

(17) **Rule Def: Definite subjects**
If DP $\rightsquigarrow$ a, a definite, and VP $\rightsquigarrow$ P, then



How would you describe the difference between **Rule Q** and **Rule Def**?

Are the following inferences valid?

(18) a. *Everything is annoyed*
Smiley is annoyed

b. *Sterny is a saxophonist*
Not everything is not a saxophonist.

c. *If everything is a saxophonist then not everything is annoyed*
Smiley is not annoyed.

Existential quantifiers

The 1-place quantifier **something** holds of any property P, just in case P has a non-empty overlap with U. Let's sketch that on the board.

(19) $[\![\textbf{something}]\!] = \{P \mid U \cap P \neq \emptyset\}$

nothing is just the negation: the set of sets that *do* have an empty intersection with U.

(20) $[\![\textbf{nothing}]\!] = \{P \mid U \cap P = \emptyset\}$

What are the following?

(21) a. $[\![\textbf{something}]\!] \cup [\![\textbf{nothing}]\!]$
b. $[\![\textbf{something}]\!] \cap [\![\textbf{nothing}]\!]$
c. $[\![\textbf{everything}]\!] \cup [\![\textbf{nothing}]\!]$
d. $[\![\textbf{something}]\!] \cup [\![\textbf{not.everything}]\!]$

Let's stick to the model on the previous page. What are the following?

(22) a. $[\![\textbf{something}]\!]^M =$
b. $[\![\textbf{nothing}]\!]^M =$

(23) a. $[\![\textbf{something}(\textbf{saxophonist})]\!]^M =$
b. $[\![\textbf{nothing}(\textbf{annoyed})]\!]^M =$

Now to map English to $\mathbf{TY}_2$:

(24) a. *something* $\rightsquigarrow$ **something**
b. *anything* $\rightsquigarrow$ **something**
c. *nothing* $\rightsquigarrow$ **nothing**

Are the following true or false? Remember we can apply **Rule Q** to any quantifier.

(25) a. *Either nothing is annoyed or something is a saxophonist*
b. *Everything is annoyed and nothing is a saxophonist.*
c. *Nothing is not a saxophonist.*

Are the following inferences valid?

(26) a. *Smiley is a saxophonist*
Something is a saxophonist

b. *Nothing is a saxophonist*
Smiley is not a saxophonist

c. $\frac{\textit{Not everything is a saxophonist}}{\textit{Something is a saxophonist}}$

d. $\frac{\textit{Everything is a saxophonist}}{\textit{Something is a saxophonist}}$

5.3.2 2-place quantifiers

We introduced the notion of 1-place quantifiers, sets of sets: for example something like $[\![Q]\!] = \{[\![\textbf{smile}]\!], [\![\textbf{happy}]\!], [\![\textbf{cat}]\!], [\![\textbf{smoke}]\!], [\![\textbf{skateboards}]\!]\}$.

Just like we can have 2-place relations (e.g., **teases**), we can also have 2-place quantifiers.

(27)

Expressions	interpreted as	Expressions	interpreted as
1-place relations	sets of individuals	1-place quantifiers	sets of properties
2-place relations	sets of pairs of individuals	2-place quantifiers	sets of ... ?
3-place relations	sets of triples of individuals	3-place quantifiers	sets of ... ?
and so on...			

Let's add a rule to the syntax of $\mathbf{TY}_2$:

(28) **New syntax rule (no. 2)**:
If Det is a 2-place quantifier, and P and Q are properties, then $Det(P)(Q)$ is a well formed formula.

And its interpretation.

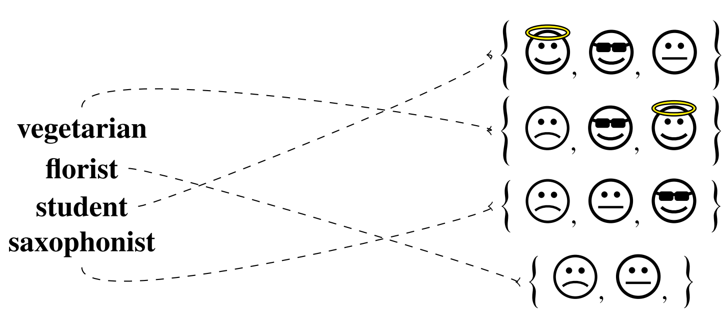
(29) **New semantics rule (no. 2)**:
If $Det(P)(Q)$ is a well formed formula, then $[\![Det(P)(Q)]\!] = \mathbf{T}$ iff $\langle [\![P]\!], [\![Q]\!] \rangle \in [\![Det]\!]$.

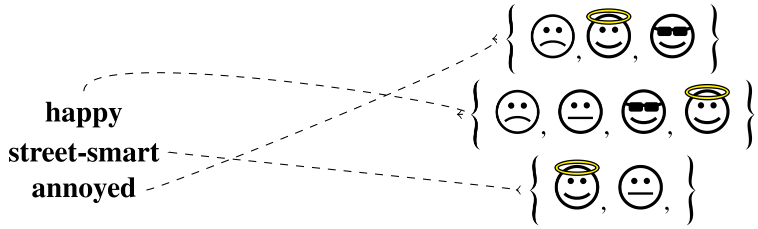
The big breakthrough of Barwise and Cooper 1981: natural language determiners like *every*, *some*, *any*, *most*, *exactly three*, are interpreted as 2-place quantfiers.

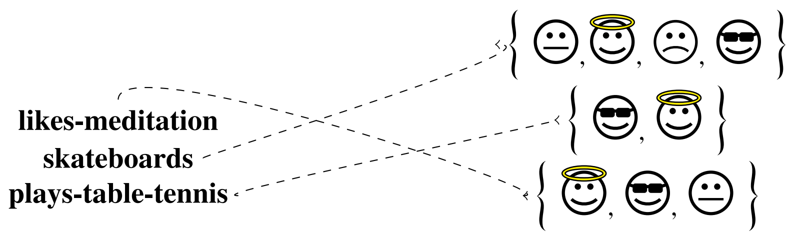
First let's add some metalanguage terms:

(30) a. $[\![\textbf{every}]\!] = \{\langle P, Q \rangle \mid P \subseteq Q\}$
b. $[\![\textbf{some}]\!] = \{\langle P, Q \rangle \mid P \cap Q \neq \emptyset\}$
c. $[\![\textbf{no}]\!] =$
d. $[\![\textbf{not.every}]\!] =$

Here's a new, bigger model







A worked example:

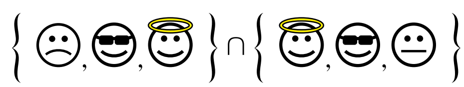
(31) Is **some**(**vegetarian**)(**likes-meditation**) true or false?

$[\![\textbf{some}(\textbf{vegetarian})(\textbf{likes-meditation})]\!] = \textbf{T}$ via (29)

. iff $\langle [\![\textbf{veg.}]\!], [\![\textbf{med.}]\!] \rangle \in [\![\textbf{some}]\!]$

$= \langle [\![\textbf{veg.}]\!], [\![\textbf{med.}]\!] \rangle \in \{\langle P, Q \rangle \mid P \cap Q \neq \emptyset\}$ via def. of $[\![\textbf{some}]\!]$

$= [\![\textbf{veg.}]\!] \cap [\![\textbf{med.}]\!] \neq \emptyset$ via def. of $\in$

$=$  $\neq \emptyset$ via def. of $[\![\textbf{veg.}]\!], [\![\textbf{med.}]\!]$

Are the following true or false?

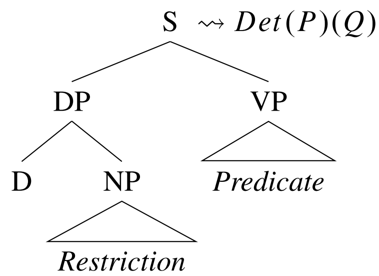
(32) a. **every**(**florist**)(**happy**)
b. **not.every**(**vegetarian**)(**skateboards**) $\vee$ **some**(**student**)(**plays-table-tennis**)
c. **no**(**florist**)(**annoyed**) $\wedge$ **some**(**saxophonist**)(**skateboards**)
d. $\neg$**no**(**student**)(**happy**)

We need a rule for interpreting natural language determiners.

(33) a. *every* $\rightsquigarrow$ **every**
b. *some* $\rightsquigarrow$ **some**
c. *any* $\rightsquigarrow$ **some**
d. *no* $\rightsquigarrow$ **no**
e. *not every* $\rightsquigarrow$ **not.every**

Plus a new rule for connecting subject quantifiers with the predicate.

(34) **Rule D: Subjects with quantificational determiners**
If D $\rightsquigarrow Det$, NP $\rightsquigarrow P$ and VP $\rightsquigarrow Q$, then,



Are the following true or false relative to the above model?

(35)
a. It's not the case that any vegetarian skateboards.
b. Every individual who skateboards is a florist.
c. Not every happy individual plays table tennis.
d. If any student likes meditation, then no florist is annoyed.

5.4 Introducing cardinalities

So far all our definitions of determiners like **every** have been using set-theoretic operators like $\cap, \cup, \subseteq$.

But we can use cardinalities to *explode* our possible definitions for quantifiers.

(36) **Cardinalities**:
If A is a set, $|A|$ = the number n such that A has n members.

Here's a definition of **most**, what does it say in prose?

(37) $[\![\textbf{most}]\!] = \{\langle A, B\rangle \mid |A \cap B| > |A - B|\}$

Propose values for the following:

(38)
a. $[\![\textbf{exactly.three}]\!] =$
b. $[\![\textbf{at.least.three}]\!] =$
c. $[\![\textbf{at.most.three}]\!] =$
d. $[\![\textbf{more.than.four}]\!] =$
e. $[\![\textbf{less.than.half}]\!] =$
f. $[\![\textbf{between.five.and.ten}]\!] =$

Assuming the English expressions have the obvious translations, are the following true or false relative to the model above?

(39)
a. *Exactly two florists are saxophonists.*
b. *Exactly one saxophonist plays table tennis.*
c. *Most vegetarians like meditation.*

(40)
a. What's the difference between **some** and **exactly.one**?
b. What's the difference between **no** and **exactly.zero**?

(41) What is $[\![\textbf{at.least.zero}]\!] =$

(42) Larry David (playing Bernie Sanders): When I ran for senator in Vermont, I got 50 percent of the black vote. His name was Marcus. (Saturday Night Live, 2015)

(43) Tips for safe, healthy tanning: Always sit at least 100 yards from the sun (The Onion, 2017)

5.5 What is *most*?

5.5.1 *most* or *more than 50%*?

Adapted from Potts (130A, 2015) lecture notes: Mark Lieberman at http://languagelog.ldc.upenn.edu/nll/?p=25 observed the following exchange:

> **Kurt Andersen**: I- I read somewhere that you said that now m- most of your audience, you believe, reads you not in English. They are not only overseas but people not in the United Kingdom or Australia. It's- it's people reading in-
>
> **John Irving**: I wouldn't say- I wouldn't say "most" but I'd say "more than half". Sure, more than half, definitely. I mean I- I sell more books in Germany than I do in the U.S. Uh I s- sell almost as many uh books in- in the Netherlands as I do in the- in the U.S.

Lots of readers left comments on Liberman's post articulating their assumptions about what *most* means, and he collected them in a follow-up (http://languagelog.ldc.upenn.edu/nll/?p=2511:

(44) a. I think 'most' licenses a default generalization, relative to a bunch of pragmatic factors.
b. I think 'most' has a normative or qualitative sense in addition to a quantitative sense.
c. For me too, "most" has a defeasible implicature of "much more than a majority".
d. I would be with John Irving - 51% of a population isn't "most" but around 60-75% would be. (90% or more would be "almost all"; well, until it hit "all" at 100%; and 75-90% would be "a very large majority").
e. "Most X are Y", to me, means a substantial majority of X are Y – certainly more than 50%-plus-1. Even two-thirds feels borderline.
f. Most has always meant "more than half (but less than all)" to me. If there are 100 of us and I say "Most of us stayed behind" I mean between 51 and 99.

Liberman looked at some dictionaries:

(45) a. *OED*: modifying a plural count noun: the greatest number of; the majority of
b. *Merriam-Webster*: the majority of
c. *American Heritage*: in the greatest quantity, amount, measure, degree, or number: to win the most votes

Here are some different $\mathbf{TY}_2$ determiners (including some context dependency):

(46) a. $[\![{}^{>}\textbf{half}]\!] = \{\langle A,B\rangle \mid \frac{|A\cap B|}{|A|} > \frac{1}{2}\}$
$= \{\langle A,B\rangle \mid |A\cap B| > |A-B|\}$
b. $[\![{}^{>}\textbf{half+}]\!] = \{\langle A,B\rangle \mid \frac{|A\cap B|}{|A|} > f\}$, where $f \gg \frac{1}{2}$
c. $[\![\textbf{plur}]\!] = \{\langle A,B\rangle \mid |A\cap B| > |C\cap B|\}$, for any C such that C stands in contrast to A.

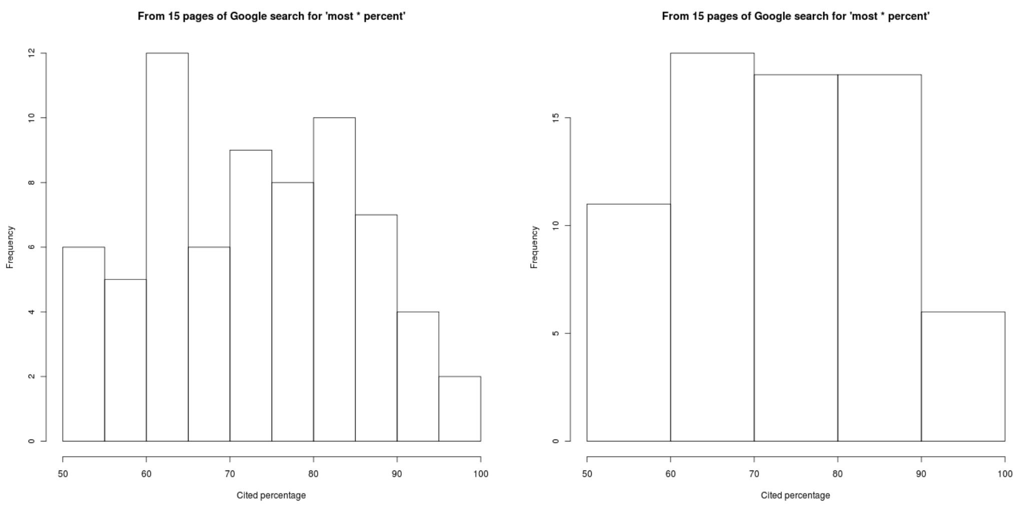
So what is English *most*? One of the following? Ambiguous between some or all of the following?

(47) *most* $\leadsto {}^{>}\textbf{half}$
$\leadsto {}^{>}\textbf{half}+$
$\leadsto \textbf{plur}$

Liberman conducted an experiment using Google, by searching for "most * percent".

picking out the first 150 with numerical percentages given, and then summarizing the distribution of those percentages with two histogram showing different ways of binning the data:

(48)



Binning by 5% increments — Binning by 10% increments

Mark Liberman's Google data on the percentages given to clarify most statements. The bimodal distribution on the left might be an artifact of the binning procedure; the plot at right suggests constant use from 60-90%.

Liberman's conclusion: "it's pretty clear that the whole range from 50.1 to 99.9 is getting some action."

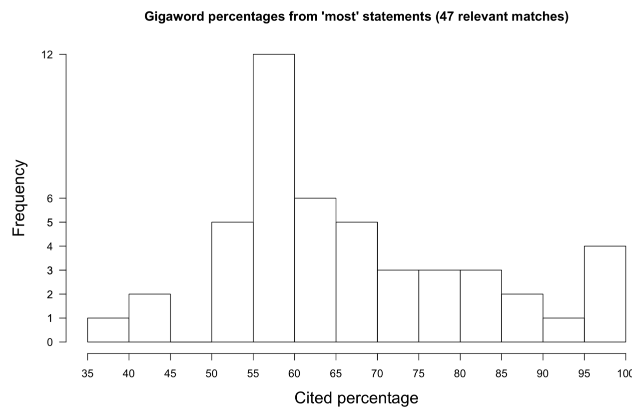
Christopher Potts aimed to replicate Liberman's study. Rather than using Google, he used *the English Gigaword*, a 1 billion word corpus of English newswire text. The regular expression:

(49)

```
(?:^|[.!?])                # Sentence/line boundary
(
    .*?                    # Try to keep the sentence context.
    \bmost                 # ‘most’ preceded by a word boudary.
    \s+
    (?:[\w\-\’]+\s+){1,4}  # Allow up to 4 word-like things here.
    \(                     # Opening parenthesis character.
    [\d.]+                 # The percentage.
    (?:\s+percent|%)
    \)                     # Closing parenthesis character.
    .*?                    # Try to keep the rest of the sentence.
)
(?:[.!?]|$)                # Sentence/line boundary.
```

How similar are Potts' results to Liberman's?

(50)



The three cases where $n < 50$:

(51) a. most homes (39 percent) have a separate room where the pc is
b. found that most of them (42 percent) focus on what he dubs
c. most of the country (42 percent) will

Only one of our three possible analyses of *most* can account for these, which one? Is this plausible?

Liberman did an additional post giving lots of citations and abstracts for psycholinguistic and theoretical work on most: http://languagelog.ldc.upenn.edu/nll/?p=2516. Of particular interest: Pietroski et al. 2009.

5.5.2 Assessing equivalence

Pietroski et al. 2009 suggest two definitions for *most*.

(52) $[\![\textbf{one.to.one}]\!] = \{\langle A, B\rangle \mid$ there is a bijective function $f : A \mapsto B\}$
(i.e., every member of A maps to a different member of B with no leftovers)

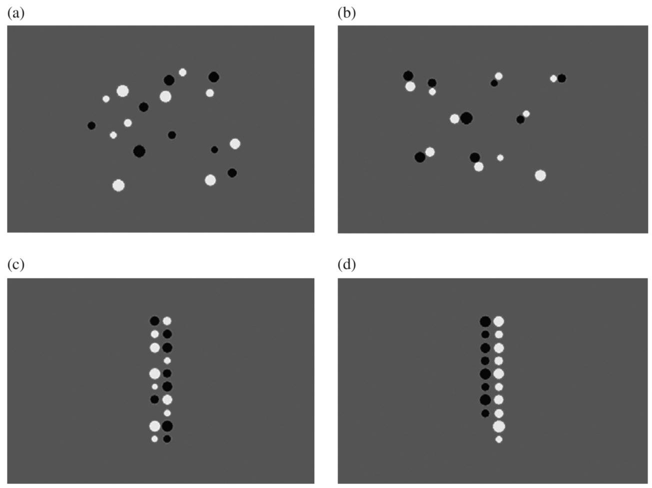
Of course $[\![\textbf{one.to.one}(A)(B)]\!] = \textbf{T}$ just in case $|[\![A]\!]| = |[\![B]\!]|$.

Pietroski et al. 2009's two readings for *most* (assume A and B are finite).

(53) a. $[\![{}^{>}\textbf{half}]\!] = \{\langle A, B\rangle \mid |A \cap B| > |A - B|\}$
b. $[\![\textbf{one.to.one}^{+}]\!] = \{\langle A, B\rangle \mid$ there's a $s \subset A \cap B$ s.t. $|s| = |A - B|\}$

> The two procedures we focus on, cardinality-comparison and OneToOnePlus-assessment, can never disagree: there is no conceivable scenario in which these algorithms yield different results. Of course, actual attempts to execute the algorithms may fail in different ways in different circumstances. But taking the outputs to be conditional specifications of truth values, for any instance of 'Most As are Bs', the two procedures cannot ever yield specifications that specify different truth values.
>
> Nonetheless, the 'truth-procedures' differ. One can imagine [individuals] who cannot represent cardinalities, and so cannot associate [Most As are Bs] with the first procedure. Likewise, one can imagine [individuals] who lack the representational resources to associate [Most As are Bs] with the second procedure. One can also imagine creatures who have the cognitive resources required to associate [Most As are Bs] with either

procedure, but in fact understand [Most As are Bs] in exactly one way, sometimes using the other procedure as a verification strategy. (Pietroski et al. 2009: 561)



(54)

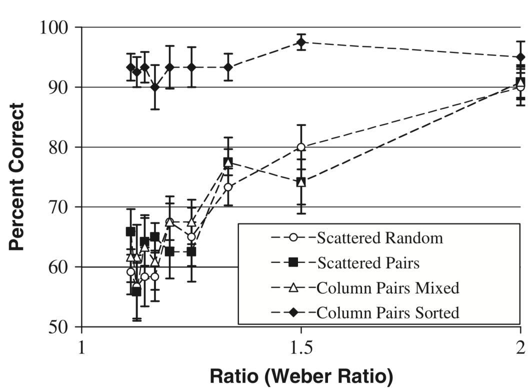
(55) (a) Scattered Random
(b) Scattered Pairs
(c) Column Pairs Mixed
(d) Column Pairs Sorted

Participants were asked to evaluate the truth of statements like 'Most of the dots are yellow' with the above contexts. The contexts were a significant predictor of accuracy:

(56)

Trial types	p
Scattered Random-Scattered Pairs	.651
Scattered Random-Column Pairs Mixed	.518
Scattered Pairs-Column Pairs Mixed	.728
Column Pairs Sorted-Scattered Random	.0001
Column Pairs Sorted-Scattered Pairs	.0001
Column Pairs Sorted-Column Pairs Mixed	.0001

As the ratio of yellow dots to blue dots increased, accuracy for the Column Pairs Sorted condition remained high, and increased from low to high for all other conditions.



(57)

Pietroski et al. consider multiple explanations for their data:

> While participants rely on the representations of the ANS on Scattered Random, Scattered Pairs, and Column Pairs Mixed trials, performance on Column Pairs Sorted trials suggests a different process altogether. ...
>
> ... participants could attend only the length of the column to reach their decision, ignoring however many dots it took to make the column, and translate their judgment of 'longer blue column' into a 'more blue dots than yellow dots' answer without error.

Big question: even with equivalent representations like (53), should we include the *algorithm used for verification* within our *semantic representations*?

What does our choice here say about our grammatical theories for, e.g.,

- people with impaired numeric abilities
- communities without numeracy
- communities with different sorts of numeric systems

5.6 Properties of quantifiers

Where **FOL** was not expressive enough to analyze the English quantifier system, $\mathbf{TY}_2$ might be too powerful.

The sky's the limit with the kinds of 2-place quantifiers we can propose:

(58) $[\![\textbf{bloop}]\!] = \{\langle A, B\rangle \mid (|A|/|B|)+\text{the current temperature in F}^0 = 50\}$

An aspect of GQ-theory is to narrow down the space of possible quantifiers in human language (see Barwise and Cooper 1981, Keenan and Stavi 1986, Keenan 1996, etc.)

5.6.1 Conservativity

(59) **Conservativity**:
A Det is *conservative* iff for all sets A, B, $Det(A)(B) = Det(A)(A \cap B)$

(60) **Universal 1** (hypothesis):
Every determiner in every natural language is conservative. (Barwise and Cooper 1981)

Basically the intuition is: to verify the truth of a $Det(A)(B)$ sentence (where Det is conservative), you only need to look at the As.

Von Fintel and Matthewson 2008 give the following intuitive way to check conservativity:

(61) a. Every man smokes $\leftrightarrow$ every man is a man who smokes.
b. Some man smokes $\leftrightarrow$ some man is a man who smokes.
c. No man smokes $\leftrightarrow$ no man is a man who smokes.
d. Most men smoke $\leftrightarrow$ most men are men who smoke.
e. Few men smoke $\leftrightarrow$ few men are men who smoke.
f. Many men smoke $\leftrightarrow$ many men are men who smoke.

The big question: are there non-conservative determiners in any natural language? The view in Keenan 1996:

> All putative counterexamples to Conservativity in the literature are ones in which a sentence of the form "*D As are B*" is interpreted as $Det(B)(A)$, where Det is conservative. So the problem is not that D fails to be conservative, rather it lies with matching the Noun and Predicate properties with the arguments of the D denotation. (Keenan 1996:63)

Potential counterexample # 1 – *only*

(62) Only dogs bark.

What does Von Fintel and Matthewson's test reveal?

A plausible denotation for *only* $\rightsquigarrow$ **only**.

(63) $[\![\textbf{only}]\!] = \{\langle A, B\rangle \mid B \subseteq A\}$

How could you state the truth conditions of "only dogs bark" given (63)?

Let's explain why **only** in (63) is non-conservative.

A way out: the proposed generalization is "every <u>determiner</u> is conservative." But what is *only*?

(64) a. Smiley only likes beans.
b. Only Smiley likes beans.
c. Only some saxophonists like beans.
d. Skateboarders only!
e. Smiley eats beans only to make everyone happy.

Potential counterexample #2: (certain readings of) *many*.

(65) Many Scandinavians have won the Nobel Prize in literature. (from Westerståhl 1985, Partee 1989)

What does Von Fintel and Matthewson's test reveal? Many Scandinavians have won the Nobel Prize. $\neq$ Many Scandinavians are Nobel Prize winning Scandinavians.

Two proposed definitions of *many* are conservative, but seem to predict that the intuitively true (65) is *false*.

(66) *Definition 1* (cardinal):
$[\![\textbf{many}_c]\!] = \{\langle A, B\rangle \mid |A \cap B| > k\}$, where k is a 'large' number.

(67) *Definition 2* (proportional):
$[\![\textbf{many}_p]\!] = \{\langle A, B\rangle \mid \frac{|A\cap B|}{|A|} > k\}$, where k is a 'large' number $\in [0, 1]$.

How would you paraphrase either reading as applied to (65)?

Westerståhl 1985: as of 1984, 14 of 81 Nobel Prize winners for literature were Scandinavian, and there are 20 million Scandinavians total. So what does each above reading of *many* predict for (65)?

Westerståhl proposes a third reading *reverse proportional*:

(68) *Definition 3* (reverse-proportional):
$[\![\textbf{many}_{rp}]\!] = \{\langle A, B\rangle \mid \frac{|A \cap B|}{|B|} > k\}$, where k is a 'large' number $\in [0, 1]$.

What equality has to hold for $\textbf{many}_{rp}$ to be conservative?

Explain why this equality need not hold.

See Cohen 2001, Romero 2015, Lauer and Nadathur 2018 for some attempts to eliminate $\textbf{many}_{rp}$.

5.6.2 Intersectivity

(69) **Intersectivity**:
A Det is *Intersective* iff for all sets A, B, $Det(A)(B) = Det(B)(A)$

Based on our definitions, *some* is intersective, but *every* is not.

(70) a. $[\![\textbf{some}]\!] = \{\langle A, B\rangle \mid A \cap B \neq \emptyset\}$
b. $[\![\textbf{every}]\!] = \{\langle A, B\rangle \mid A \subseteq B\}$

Given our above definitions, what equalities need to hold if *some* and *every* are intersective?

What about *no*?

Let's look at *not every* in more depth.

(71) $[\![\textbf{not every}]\!] = \{\langle A, B\rangle \mid A \not\subseteq B\}$

Provide a pair of English sentences that supports the classification as intersective or not intersective. Use arrows indicating entailment relations do and do not hold.

Keenan 1996 proposes that *intersectivity* is relevant for acceptability in "existential *there*" sentences in English (see Milsark 1977).[2]

(72) Keenan's Generalization: If an NP is acceptable in an "existential *there*" sentence, then it is intersective.

How does Keenan's generalization fare with the following data (from Keenan 1996)?

[2] Though see Keenan 2003 where he says *intersectivity* is sufficient but not necessary for existential sentences.

(73) a. There wasn't more than one student at the party
b. Are there more dogs than cats in the garden? .
c. There was no one but John in the building at the time
d. Weren't there more male than female students at the party?

(74) a. *There wasn't John at the party.
b. *There were most students on the lawn.
c. *Was there every student in the garden?
d. *There wasn't every student but John in the garden
e. *Were there two out of three students in the garden?
f. *There weren't John's ten students at the party

5.6.3 Monotonicity

The most famous quantifier property.

(75) **Monotonicity**

a. A function Q is upward entailing (upward monotone) iff wherever $A \subseteq B$, then $\mathsf{Q}(A) \rightarrow \mathsf{Q}(B)$
b. A function Q is downward entailing (downward monotone) iff wherever $A \subseteq B$, then $\mathsf{Q}(B) \rightarrow \mathsf{Q}(A)$
c. A function Q is non-monotone if it is neither increasing nor decreasing.

Remember our lexical entailment $\sqsubseteq$, such that *waltz* $\sqsubseteq$ *dance*? Are the following DPs upward monotone, downward monotone, or non-monotone?

(76) a. *Everyone*
b. *Someone*
c. *No-one*
d. *Exactly three students*

(77) **Monotonicity** (for determiners)

a. A determiner Q is *upward entailing on its right argument* iff wherever $A \subseteq B$, then $Det(A)(C) \rightarrow Det(B)(C)$
b. A determiner Q is *upward entailing on its left argument* iff wherever $A \subseteq B$, then $Det(C)(A) \rightarrow Det(C)(B)$
c. A determiner Q is *downward entailing on its right argument* iff wherever $A \subseteq B$, then $Det(B)(C) \rightarrow Det(A)(C)$
d. A determiner Q is *downward entailing on its left argument* iff wherever $A \subseteq B$, then $Det(C)(B) \rightarrow Det(C)(A)$

What are the monotonicity properties of the following determiners?

(78) a. *some* ( ) ( )
b. *no* ( ) ( )
c. *every* ( ) ( )
d. *at most ten* ( ) ( )
e. *few* ( ) ( )
f. *exactly three* ( ) ( )

g. *most* ( ) ( )

The English adverbial particle *ever* has a highly restricted distribution. On the basis of the following examples (where * marks ungrammatical cases, as usual), let's formulate a generalization in terms of the monotonicity properties of determiners about where *ever* can appear:

(79)
a. No [$_{NP}$ students who have **ever** taken semantics] [$_{VP}$ have been to Peru]
b. No [$_{NP}$ students] [$_{VP}$ have **ever** been to Peru]
c. *Some [$_{NP}$ students who have **ever** taken semantics] [$_{VP}$ have been to Peru]
d. *Some [$_{NP}$ students] [$_{VP}$ have **ever** been to Peru]
e. At most three [$_{NP}$ students who have **ever** taken semantics] [$_{VP}$ have been to Peru]
f. At most three [$_{NP}$ students] [$_{VP}$ have **ever** been to Peru]
g. *Exactly three [$_{NP}$ students who have **ever** taken semantics] [$_{VP}$ have been to Peru]
h. *Exactly three [$_{NP}$ students] [$_{VP}$ have **ever** been to Peru]
i. Every [$_{NP}$ student who has **ever** taken semantics] [$_{VP}$ has been to Peru]
j. *Every [$_{NP}$ student] [$_{VP}$ has **ever** been to Peru]

ever is a 'negative polarity item' (or NPI), like *any*, *a red cent*, *a rat's ass*, *budge*, *much (water/evidence/...)*, and many others.

The distribution of NPIs is a famous problem in syntax-semantics. The data in (79) support one famous theory from Fauconnier 1975, 1978 and Ladusaw 1979.

(80) *The Ladusaw-Fauconnier Generalization*:
Negative polarity items occur within arguments of monotonic decreasing functions but not within arguments of monotonic increasing functions.

Though there is a ton of later work disputing and/or refining this generalization (see Barker 2018; Israel 2011; Zwarts 1996, 1998, and many others).

5.7 Possible Paper Topics

- Barker 2018 presents a theory of negative polarity items (NPIs). These are quantifiers like *any* which are licensed underneath negation. Could it be extended to more empirical phenomena. For example, Zwarts 1998 proposes that different NPIs have different strengths with different licensing conditions (e.g., *any at all* appears in fewer environments than *any*).
- What about positive polarity items, like *some*, which <u>cannot</u> appear in the scope of negation? See Szabolcsi 2004 for example, or Mayraz 2018 for a cross-linguistic take.
- Pietroski et al. 2009's proposal that determiner meanings should reflect the online algorithm for calculating truth is provocative and wide-ranging. Basically it says that determiner meanings should be tightly linked to the interpreters' competence with respect to quantification. What does this say about communities which lack numeric systems (see Frank 2012), or individuals with poor numeric abilities, or children in early stages of acquiring numerosity?
- There have been a few generalizations about what can appear in existential pivots (Keenan 1987, 2003). But there are other environments which seem to like intersective quantifiers, for example languages which have differential object marking like Turkish (Enc 1991) or Persian (Jasbi 2016), or English relational *have* (e.g., *I have a/*every friend*).

- This handout focuses on 1-place and 2-place quantifiers. But there are plausible candidates for $\geq$3-place quantifiers (see Keenan 1996; Westerståhl 2015), like those which include *except* or *unless*, or ones which incorporate adjectives, like *more* ADJ_1 *than* ADJ_2 *NPs*. What is the syntax and semantics of these and how do they figure into the theories discussed here?
- Westerståhl 2015 discusses a *pyramid* notation for representing GQ meanings which goes some way in explaining restrictions on GQ meanings. Determiner meanings don't express the relationship between set A and set B, rather the relationship between $A \cap B$ and $A - B$. Does this theory of GQs help us understand their behavior, acquisition, processing, cross-linguistic distribution etc.?

5.8 Further Reading

- The chapter in Partee, Meulen, and Wall 1990 on GQs is excellent, as is Westerståhl 2015 chapter on GQs.
- Steinert-Threlkeld and Szymanik 2019 explain the link between determiners and their universal properties (e.g., conservativity) in terms of learnability.
- Frank et al. 2008, Frank 2012 explore the link between an individuals numeric capability and their understanding of quantificational language, looking at the Pirahã, an Amazonian community with a limited numerical system. See Nadathur 2016 for a discussion of the implications.
- The semantics of *only* is another huge topic in semantics, see Rooth 1996, Beaver and Clark 2003, Horn 2009, Erlewine 2014, and many others.
- Another big topic not adequately covered here: negative concord (e.g., *I ain't got nothing to say* in certain English dialects, as well as Spanish, Hebrew, Italian). See De Swart and Sag 2002 and Zeijlstra 2004 for starting points.

Bibliography

Barker, Chris. 2018. "Negative polarity as scope marking". *Linguistics and Philosophy* 41 (5): 1–28.

Barwise, Jon, and Robin Cooper. 1981. "Generalized quantifiers and natural language". *Linguistics and Philosophy* 4 (4): 159–219.

Beaver, David I., and Brady Zack Clark. 2003. "*Always* and *Only*: Why not all focus sensitive operators are alike". *Natural Language Semantics* 11 (4): 323–362.

Cohen, Ariel. 2001. "Relative readings of *many*, *often* and generics". *Natural Language Semantics* 69:41–67.

De Swart, Henriette, and Ivan A. Sag. 2002. "Negation and negative concord in Romance". *Linguistics and Philosophy* 25 (4): 373–417.

Enc, Mürvet. 1991. "The semantics of specificity". *Linguistic Inquiry* 22 (1): 1–25.

Erlewine, Michael Yoshitaka. 2014. "Movement out of focus". PhD thesis, MIT.

Fauconnier, Gilles. 1975. "Polarity and the scale principle". In *The Proceedings of the Chicago Linguistics Society 11*, edited by R. E. Grossman, L. J. San, and T.J. Vance, 188–199. Chicago, IL: Chicago Linguistics Society.

— . 1978. "Implication reversal in a natural language". In *Formal Semantics and Pragmatics for Natural Languages*, edited by F. Guenther and S. J. Schmidt, 289–301. Dordrecht: Reidel.

Frank, Michael C. 2012. "Cross-cultural differences in representations and routines for exact number". In *Melanesian Languages on the Edge of Asia: Challenges for the 21st Century*, edited by Nicholas Evans and Marian Klamer, 219–238. University of Hawai'i Press.

Frank, Michael C., et al. 2008. "Number as a cognitive technology: Evidence from Pirahã language and cognition". *Cognition* 108:819–824.

Gallin, Daniel. 1975. *Intensional and Higher Order Modal Logic*. Amsterdam: North-Holland.

Horn, Laurence R. 2009. "ONLY XL: The assertoric asymmetry of exponibles". In *Proceedings of Semantics and Linguistic Theory 19*, edited by Ed Cormany, Satoshi Ito, and David Lutz, 198–221. Ithaca, NY: CLC Publications.

Israel, Michael. 2011. *The Grammar of Polarity*. Cambridge: Cambridge University Press.

Jasbi, Masoud. 2016. "Three types of indefinites in Persian: Simple, complex, and antidefinite". In *The Proceedings of Semantics and Linguistic Theory 26*, 244–263. Ithaca, NY: CLC Publications.

Keenan, Edward L. 1987. "A semantic definition of "indefinite NP"". In *The Representation of (In)definiteness*, edited by Eric Reuland and Alice ter Meulen, 286–317. Cambridge, MA: MIT Press.

— . 1996. "The semantics of determiners". In *The Handbook of Contemporary Semantic Theory*, edited by Shalom Lappin, 41–63. Oxford: Blackwell.

— . 2003. "The definiteness effect: Semantics or pragmatics?" *Natural Language Semantics* 11 (2): 187–216.

Keenan, Edward L., and Jonathan Stavi. 1986. "A semantic characterization of natural language determiners". *Linguistics and Philosophy* 9:253–326.

Ladusaw, William. 1979. "Polarity sensitivity as inherent scope relations". PhD thesis, University of Texas, Austin.

Lauer, Sven, and Prerna Nadathur. 2018. "Quantified indicative conditionals and the relative reading of *most*". In *The Proceedings of Sinn und Bedeutung 21*, 767–784. Edinburgh: University of Edinburgh.

Mayraz, Omri. 2018. "Inverse-scope and semi-PPIs: Evidence from Hebrew". Paper presented at *North East Linguistics Society 49*.

Milsark, Gary. 1977. "Toward an explanation of certain pecularities in the existential construction in English". *Linguistic Analysis* 3:1–30.

Nadathur, Prerna. 2016. "Language, perception, and thought: What can neo-Whorfianism teach us about cognition?" Ms., Stanford University.

Partee, Barbara B. H. 1989. "*Many* quantifiers". In *The Proceedings of the 5th Eastern States Conference on Linguistics*, 383–402. Columbus, OH: Ohio State University.

Partee, Barbara B. H., Alice G. ter Meulen, and Robert Wall. 1990. *Mathematical Methods in Linguistics*. Dordrecht: Springer.

Pietroski, Paul, et al. 2009. "The meaning of *most*: Semantics, numerosity and psychology". *Mind and Language* 24 (5): 554–585.

Romero, Maribel. 2015. "The conservativity of *many*". In *The Proceedings of the Twentieth Amsterdam Colloqium*, 20–29. Amsterdam: University of Amsterdam.

Rooth, Mats. 1996. "Focus". In *Handbook of Contemporary Semantic Theory*, edited by Shalom Lappin, 271–298. London: Blackwell.

Steinert-Threlkeld, Shane, and Jakub Szymanik. 2019. "Learnability and Semantic Universals". To appear in *Semantics and Pragmatics*.

Szabolcsi, Anna. 2004. "Positive polarity–negative polarity". *Natural Language and Linguistic Theory* 22 (2): 409–452.

Von Fintel, Kai, and Lisa Matthewson. 2008. "Universals in semantics". *The Linguistic Review* 25 (1-2): 139–201.

Westerståhl, Dag. 1985. "Logical constants in quantifier languages". *Linguistics and Philosophy* 8:387–413.

— . 2015. "Generalized quantifiers in natural language semantics". In *The Handbook of Contemporary Semantic Theory*, edited by Shalom Lappin and Chris Fox, 9–39. West-Sussex: Wiley-Blackwell.

Zeijlstra, Hedde. 2004. "Sentential negation and negative concord". PhD thesis, University of Amsterdam.

Zwarts, Frans. 1996. "A hierarchy of negative expressions". In *Negation: A Notion in Focus*, edited by H. Wansing, 169–194. Berlin/New York: Walter de Gruyter.

— . 1998. "Three types of polarity". In *Plurality and Quantification*, edited by F. Hamm and E. W. Hinrichs, 177–238. Alphen aan den Rijn: Kluwer.