9. Formalizing scalar reasoning

9.1 Introduction

Here is our basic recipe for calculating a scalar implicature.

(1) $U =$ 'Some of the puppies escaped.'
$p =$ *not all the puppies escaped*

a. Contextual premise: Sp knows how many puppies escaped
b. Contextual premise: Kyle is cooperative.
c. By (b), Kyle's utterance will be maximally informative, relevant, and true.
d. The utterance "All the puppies escaped" (= U') is more informative/relevant than U.
e. By (b–d), Sp must lack evidence that U' is true.
f. By (a) and (e), Kyle lacks evidence for U' because it is false.
g. Therefore, p (it is false that all of the puppies escaped)

Scalar implicatures like (1) involve:

a. A clash between quality and quantity (Sp is being as informative as truth allows).
b. Reasoning about a scale of expressions (e.g., $\langle some, all \rangle$

The goal for this handout is to

a. flesh out a theory of scalar implicatures
b. explore theories of pragmatic reasoning using probabilistic game theory

9.2 Scalar implicatures

Scalar implicatures imply the use of a scale.

Some scales can be clearly extrapolated from asymmetrical entailment, e.g., *some* vs. *all* (assuming existential import). But this isn't always the case:

(2) A: Did you get to Boston?
B: We got to Vegas.

(3) A: Do you speak Portuguese?
B: My husband does.

(4) A: Did you mail the letter?
B: I wrote it.

None of the above involve asymmetrical entailment, but each implies that B's answer is the most informative true thing B can say, implying some kind of *ad-hoc* scale.

Hirschberg 1985: The context could make available any sort of *partially ordered scale* (roughly, a scale which allows ties). For (2) something like the following:

(5) $Sacramento \prec Denver \prec Vegas \prec \{Gary, Houston\} \prec Boston$

A recurring pattern in scalar implicatures noted by Horn 1972:

(6) a. The literal meaning of U is a "lower bound reading"
b. The implicated meaning of U is a "upper bound reading".

(7) a. "Some of the puppies escaped."
Literal: At least some of the puppies escaped, maybe all.
Pragmatic: Some but not all of the puppies escaped.

(8) a. "We got to Vegas."
Literal: We got at least to Vegas maybe further.
Pragmatic: We got to Vegas and no further.

(9) a. "My husband speaks Portuguese."
Literal: ... and maybe I do too.
Pragmatic: ... and only him.

Given a contextually supplied scale like (5) or *some* $\prec$ *all*, a scalar implicature will tell you that the uttered term is **the highest term on the scale which is true**.

In the case of ties, e.g., *Gary* and *Houston* in (5), a scalar implicature will be unspecified about the truth of the tied term (Hirschberg 1985).

Here, we're going to spell out this reasoning formally, arguing that humans use *probabilistic reasoning* to draw these implicatures.

This orients pragmatics in terms of domain general cognitive abilities of humans, i.e., drawing probabilistic inferences.

9.3 Bayesian inference

The following is adapted from Scontras, Tessler, and Franke 2018.

(10) Jones has a deck of three cards. One is blue on both sides. A second is blue on one side and red on the other. A third is red on both sides: $\{B/B, B/R, R/R\}$

Jones shuffles and shows you one side of one card. It's blue.

What is the probability that the other side is blue?

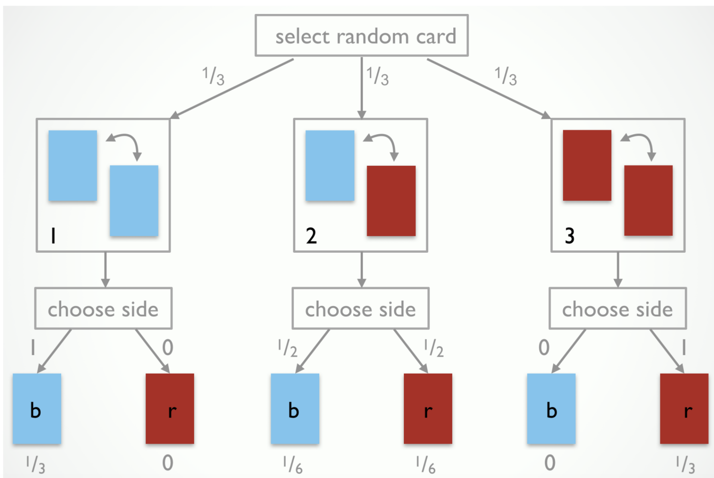
Many people might say 0.5. That would be the correct answer to the following question:

(11) John shuffles and shows you one side of the card.

What is the probability that it is blue?

But problem (10) involves an "observation" which causes us to update our knowledge mid game.

(12)



a. For each card c, $P(c) = 0.33$

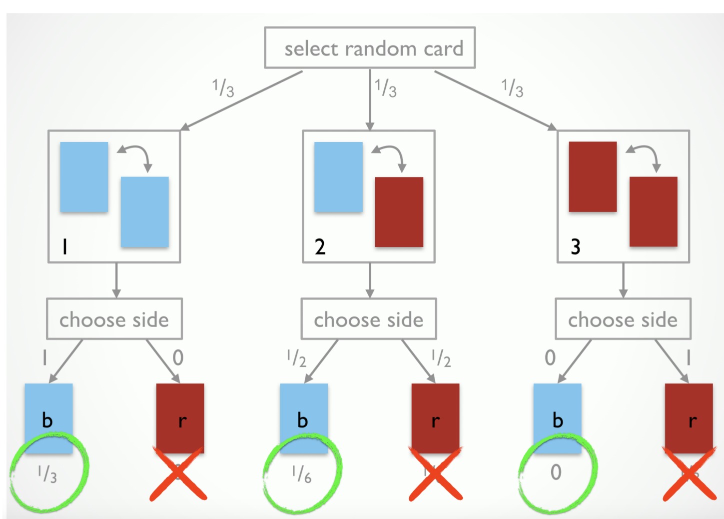
b. The likelihood of red/blue given card 1, $P(r|c1) = 0$ and $P(b|c1) = 1$

c. What about the likelihood of red/blue given card 2 and 3?

The probability of obtaining each card-color combo is calculated by multiplying down the branches (in the lowest row). Logically impossible outcomes get probability 0.

Returning to Jones, let's say we see Jones's blue side. We now want to know, did we see the blue side of $c1$, $c2$, or $c3$?

(13)



a. First we update our information state: the side is blue, so eliminate the red possibilites.
b. Intuitively, it's impossible that we saw the blue side of $c3$.
c. It is twice as likely that we saw $c1$ than $c2$.

How do we work out whether we saw the blue side of $c1$ or $c2$? The green circled probabilities correctly tell us that $c1$ is twice as likely as $c2$, but they don't add up to 1.

We need to *normalize* (in a very specific sense).

(14) **Normalize** (for us): divide a set of probabilites X by some constant so that the members of X sum to 1 (but maintain the same proportional relationships).

Let's take the constant to be $1/2$ $(1/3 + 1/6)$.

(15) a. $P(c1) = 0.33/0.5 = 0.66$
b. $P(c2) = 0.165/0.5 = 0.33$

The question (10) is basically what is $P(c1)$ which is 0.66.

This is an instance of **Bayesian Inference**, crucial in theories of learning and inference.

(16) **Probability distribution** (informal):
A function which assigns a probability for each member in a set of outcomes.

(17) **Bayesian Inference** (informal):
Assigning a probability distribution given some observation. The probability of outcome A given observation B directly relates to how likely B's occurrence is given A.

The question of *probabilistic pragmatics*: How do interlocutors update their beliefs given the speaker's utterance, i.e., how likely is the truth of meaning p given the observation of utterance U?

(18) **Probability distribution** (formal):
Let X be a set of mutually discrete outcomes, a probability distribution over X is $P : X \mapsto [0,1]$, such that $\Sigma_{x \in X} P(x) = 1$

The following give P for a combination of hair-eye color combos for your blind date.

(19) a.

	brown	blue	green
black	.4	.02	.01
blond	.1	.3	.1
red	.01	.01	.05

b. $H = \{\textit{black, blond, red}\}, E = \{\textit{brown, blue, green}\}$
c. $X = H \times E$
d. $\textbf{black} = \{\langle h, e\rangle \in X \mid h = \textit{black}\}$

What is $P(\textbf{black})$? What about $P(\textbf{green})$? NB: no normalizing required here.

Let's say we learn that our blind date has *blue* eyes. We can now work out the probability for each hair color using Bayesian inference.

(20) **Bayes Rule:**

Where A is an outcome and B is an observation:

$$P(A|B) = \frac{P(B|A) \cdot P(A)}{P(B)}$$

(21) $P(\textit{black-blue}|\textbf{blue}) = \frac{P(\textbf{blue}|\textit{black-blue}) \cdot P(\textit{black-blue})}{P(\textbf{blue})}$

$$= \frac{1 \cdot 0.02}{0.33} = \frac{0.02}{0.33} = 0.061$$

(22) $P(\textit{blond-blue}|\textbf{blue}) = \frac{P(\textbf{blue}|\textit{blond-blue}) \cdot P(\textit{blond-blue})}{P(\textbf{blue})}$

$$=$$

(23) $P(\textit{red-blue}|\textbf{blue}) =$

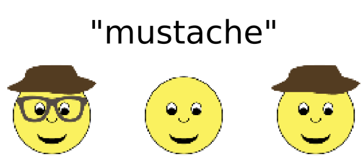
9.4 Ad-hoc implicatures

Frank and Goodman 2012 propose how Bayesian inference can be applied to scalar implicature. The following paradigm is from Stiller, Goodman, and Frank 2011 (based ultimately on Rosenberg and Cohen 1964).

Imagine someone says the utterance in quotes. Which face do you think they are referring to?

(24)

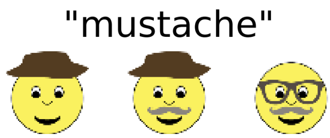

(25)


(26)


(27)


(28)


(29) 

(30) 

(31) 

The reasoning behind your choices is scalar. In each case, combinations of *hat*, *glasses*, *mustache* are forming a partially ordered scale, supplied by the context.

The choice of the referent smiley face involves reasoning about what the most informative utterance would have been given the alternatives. What would Grice say about a basic case like (27)?

(32) **Ordinary scalar implicatures** (like (27)):
U generates an ordinary/first order scalar implicature p if we calculate p relative to literal meaning of alternative(s) U'.

Cases like (30) are much trickier. What is the reasoning behind it?

(33) **Second order scalar implicatures** (like (30)):
U generates a second order implicature p if we calculate p relative to the implicatures generated by alternative(s) U'.

The precise formalization we'll learn is **Rational Speech Act** (RSA) theory.

9.5 Rational Speech Act theory

RSA is proposed in Frank and Goodman 2012, but has lots of subtly different versions focusing on different phenomenon, including

a. implicatures with multiple quantifiers (Potts et al. 2016)
b. gradable adjectives (Lassiter and Goodman 2017)
c. implicatures with conjunction/disjunction (Bergen, Levy, and Goodman 2016)
d. hyperbole (Kao, Bergen, and Goodman 2014)
e. negative antonyms, negated gradable expressions (Tessler and Franke 2018; Yoon et al. 2017)
f. social meaning (Burnett 2017; Qing and Cohn-Gordon 2019)
g. belief revision (Degen, Tessler, and Goodman 2015)
h. L1 acquisition of implicatures (Stiller, Goodman, and Frank 2011)
i. word learning (Frank and Goodman 2014)

RSA involves a reference game, where players try to associate utterances with referents, just like the faces game above.

9.5.1 Setting up a game

The game has the following ingredients:

(34) a. T, a set of referents
b. M, a set of messages
c. $[\![\cdot]\!] : M \mapsto (T\{0,1\})$, an interpretation function giving literal meanings to messages, where 0 and 1 stand for false and true.
d. $P : T \mapsto [0,1]$, the prior likelihood that each referent is chosen
e. $C : M \mapsto \mathbb{R}$, a cost function on messages

What is the set up for (27)?

(35) a. $T = \{\textbf{left}, \textbf{mid}, \textbf{right}\}$
b. $M = \{\text{'hat'}, \text{'mustache'}, \text{'glasses'}\}$
c. $[\![hat]\!](\textbf{left}) = 1, [\![hat]\!](\textbf{mid}) = 1, [\![hat]\!](\textbf{right}) = 0,$
$[\![mustache]\!](\textbf{left}) = 0, [\![mustache]\!](\textbf{mid}) = 1, [\![mustache]\!](\textbf{right}) = 0,$
$[\![glasses]\!](\textbf{left}) = 0, [\![glasses]\!](\textbf{mid}) = 0, [\![glasses]\!](\textbf{right}) = 0$
d. $P(\textbf{left}) = 0.33,\ P(\textbf{mid}) = 0.33,\ P(\textbf{right}) = 0.33$
e. $C(\text{'hat'}) = 0,\ C(\text{'mustache'}) = 0,\ C(\text{'glasses'}) = 0$

Re: (d), we have no reason to assume any face is more likely to be referred to than any other face. (d) could be a useful way to spell out a notion of *salience*, i.e., the prior likelihood that something will be referred to.

Re: (e), we're not assuming any message costs more than any other message.

A player (the speaker) will supply a message in M, and the other player (the listener) will assign a probability distribution over T corresponding to the likelihood that the speaker intended to refer to each $t \in T$.

Just like with Jones and the red/blue cards, the listener is updating his/her beliefs based on an observation (the speaker's message).

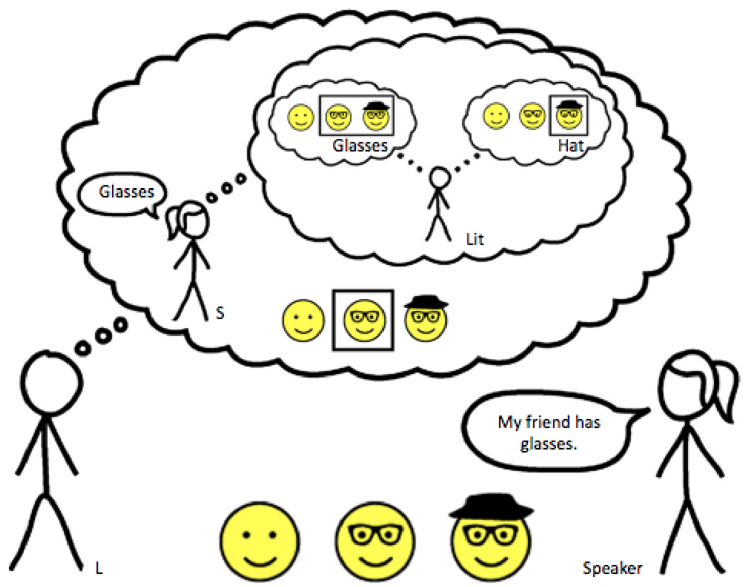
9.5.2 Modeling the players

The goal for the listener is to assign probabilities to each face-message pairing so we can make a rational decision about what the speaker intended.

Therefore, *both* players, the listener and the speaker, are modeled as probability distributions over message-face pairs.

a. for any referent r, the speaker assigns a probability to a message m (the likelihood the listener will choose correctly r given m)
b. for any message m, the listener assigns a probability to a referent r (the likelihood the speaker intended r given m).

Both the speaker and listener reason in terms of each other's behavior. For this reason, the game is recursive and can undergo unlimited iterations.

(36) 

In any case, the game must start somewhere. It starts with a *literal listener*, a probabilistic agent who does not calculate implicatures but only reasons (via basic Bayesian inference) about the literal meanings of expressions.

(37) **Literal listener** (rough):
Given a message m and referent r, a literal listener is a distribution L_0

$$L_0(r|m) = \frac{P(m|r) \cdot P(r)}{P(m)}$$

How do we spell out each term? $P(r)$, 'salience of r', is supplied by the set-up (0.33 above).

The other two terms aren't very different from the eye/hair color example.

(38) $P(m|r)$ is the likelihood of the literal message is true given the referent.

$$P(m|r) = [\![m]\!](r)$$

i.e., just the truth (1) or falsity (0)

$P(m)$, the probability of the message, is our normalizing constant, which we'll use to ensure that everything adds up to 1. It's obtained by summing up $P(m|r) \cdot P(r)$ for each value of r.

(39) **Literal listener** (expanded):
Given a message m and referent r, a literal listener is a distribution L_0

$$L_0(r|m) = \frac{[\![m]\!](r) \cdot P(r)}{\Sigma_{r' \in T} [\![m]\!](r') \cdot P(r')}$$

i.e., truth multiplied by salience, normalized (across referents)

We used a very similar formula when we calculated the card trick problem.

Given L_0, a speaker can reason about L_0's behavior, i.e., "which term should I choose, given L_0's preferences?".

(40) **Pragmatic speaker**:
Given a message m and referent r, a pragmatic speaker is a distribution S_1

$$S_1(m|r) = \frac{L_0(r|m)}{\Sigma_{m' \in M} L_0(r|m')}$$

i.e., the literal listener's behavior, normalized (across messages)

Given S_1, a pragmatic listener can reason about S_1's behavior, i.e., "which referent should I choose, given S_1's preferences?".

(41) **Pragmatic listener**:
Given a message m and referent r, a pragmatic listener is a distribution L_1

$$L_1(r|m) = \frac{S_1(m|r) \cdot P(r)}{\Sigma_{r' \in R} S_1(m|r') \cdot P(r')}$$

i.e., just like the literal listener, except we replace the truth of the utterance with S_1's behavior: S_1's behavior multiplied by salience, normalized (over referents)

This looks a bit complicated but luckily we can use code/calculators/spreadsheets, and we don't have to memorize these formulas, just copy them from the handout.

Key insight here, we can abstract S_1 and L_1 to S_n and L_n, and have an unlimited number of speakers reasoning about L_{n-1}'s behavior.

9.5.3 A basic game

Let's work through a basic scalar implicature.

(42)

Here's our set-up for this game.

(43) a. $[\![\cdot]\!] =$

	l	**m**	**r**
"hat"	1	1	0
"glasses"	0	0	0
"mustache"	0	1	0

b. for any r, $P(r) = 0.33$ (i.e., even priors/salience)

c. for any m, $C(m) = 0$ (i.e., no message is costly)

Our desired behavior for this game is that a listener will choose **l** given the message "hat", and a speaker will utter "mustache" to refer to **m**.

The literal meaning via $[\![\cdot]\!]$ does not make this prediction.

First we calculate L_0. Tip!: when the priors are even (i.e., P assigns the same value to any r), as in this game, we can just delete P from the formula.

With these tables, the *rows are always the given element*. We are modelling $L_0(r|m)$, meaning 'the listener's preference for a referent r given a message m". As the message is given, the messages should be the rows.

(44) **Literal listener**:

$L_0 =$

	l	**m**	**r**
"hat"	0.5	0.5	0
"glasses"	0	0	0
"mustache"	0	1	0

$$\frac{[\![m]\!](r)}{\Sigma_{r' \in T}[\![m]\!](r')}$$

i.e., just make sure all the rows add to 1 by dividing each term by the row total

This is a description of a literal listener's behavior, i.e., a listener who doesn't "do pragmatics".

a. On hearing "mustache" he decisively chooses **m**
b. On hearing "glasses" he rejects all three faces
c. On hearing "hat" his preference between **l** and **m** is evenly split.

Now we can calculate the pragmatic speaker S_1. Step 1, transpose L_0 to a "speaker table": the given element is the referent, so they are the rows now.

(45) L_0 transposed =

	"hat"	"glasses"	"mustache"
l	0.5	0	0
m	0.5	0	1
r	0	0	0

i.e., just swap the rows and columns, making sure the numbers swap too!

To get S_1, we just normalize by making sure the rows add to 1 (dividing by the row total).

(46) **Pragmatic speaker**:

$S_1 =$

	"hat"	"glasses"	"mustache"
l	1	0	0
m	0.33	0	0.66
r	0	0	0

$$\frac{L_0(r|m)}{\Sigma_{m' \in M} L_0(r|m')}$$

This is a description of our pragmatic speaker's behavior.

a. Observing the left face, she decisively chooses "hat".
b. Observing the right face, she doesn't choose any message.
c. Observing the middle face, she has a clear preference for "mustache"

(c) is clearly the emergence of pragmatics, a preference for "mustache" due to its higher informativity.

Now the pragmatic listener L_1. We switch back to a listener table: the message is given, so the rows are messages.

(47) S_1 transposed is

	l	**m**	**r**
"hat"	1	0.33	0
"glasses"	0	0	0
"mustache"	0	0.66	0

Don't forget to transpose the numbers too!

To get L_1, all we do is normalize by dividing by the row total. Again, we can drop the $P(r)$ term because priors are even.

(48) **Pragmatic listener**:

$L_1 =$

	l	**m**	**r**
"hat"	0.75	0.25	0
"glasses"	0	0	0
"mustache"	0	1	0

$$\frac{S_1(m|r)}{\Sigma_{m' \in M} S_1(m'|r)}$$

This is a description of a pragmatic listener

a. On hearing "hat", he has a strong preference for the left face vs. middle (implicature!)
b. On hearing "glasses" he rejects all faces.
c. On hearing "mustache" he decisively chooses the middle face.

We can iterate this reasoning, where S_n reasons about L_{n-1}, over and over again.

As n rises, L_n's preference for pairing "hat" with the left face approaches 1, and for pairing "hat" with the middle face approaches 0, deriving the (listener's) implicature.

Similarly, as n rises, S_n's preference for pairing the middle face with "mustache" approaches 1, and for pairing the middle face with "hat" approaches 0, deriving the (speaker's) implicature.

As far as this very simple game goes, there are several connections to Grice.

a. How is the interlocutors' observance of quality represented?
b. Which terms are more informative?
c. Why does the speaker prefer "mustache" for the middle face?
d. Why does the listener prefer the left face on hearing "hat"?

9.6 Salience

Intuitively, what happens if the speaker says "hat" in the following context:

(49)

Maybe there is still the implicature: "hat" implicates ¬*mustache*, but the effect is weaker.

Is there a Gricean explanation for this (see Clark, Schreuder, and Buttrick 1983 on the notion of contextual "salience")?

RSA has an inbuilt way of representing salience: the prior probability of referring to r (see Collins and Jasbi 2013; Frank and Goodman 2012). We simply update $P(r)$.

(50) a. $[\![\cdot]\!] =$

	l	**m**	**r**
"hat"	1	1	0
"glasses"	0	0	0
"mustache"	0	1	0

b. $P =$

	P
l	0.1
m	0.8 (more salient!)
r	0.1

c. for any m, $C(m) = 0$ (i.e., no message is costly)

Let's crunch through the reasoning, and see the effect on the implicature. As priors are not even, they can't be deleted from the listener formulas anymore.

(51) **Literal listener**:

$L_0 =$

	l	**m**	**r**
"hat"	0.11	0.89	0
"glasses"	0	0	0
"mustache"	0	1	0

$$\frac{[\![m]\!](r) \cdot P(r)}{\Sigma_{r' \in T} [\![m]\!](r') \cdot P(r')}$$

truth (0 or 1) multiplied by salience (0.1 or 0.8), divided by the row total.

a. On hearing "hat", the literal listener has a strong preference for choosing the middle head (cf. the last game).

b. On hearing "glasses", the listener rejects each face (as before)

c. On hearing "mustache", the listener decisively chooses the middle face (as before)

As before, to get S_1, transpose, then divide by row total.

(52) L_0 transposed

	"hat"	"glasses"	"mustache"
l	0.11	0	0
m	0.89	0	1
r	0	0	0

(53) **Pragmatic speaker** (divide each number above by row total):

$S_1 =$

	"hat"	"glasses"	"mustache"
l	1	0	0
m	0.47	0	0.53
r	0	0	0

$$\frac{L_0(r|m)}{\Sigma_{m' \in M} L_0(r|m')}$$

a. On observing left face, S_1 chooses "hat"

b. On observing middle face, $S_!$ is evenly split between "hat" and "mustache"

c. On observing right face, S_1 says nothing

Why do we get result (b) for the speaker?

Now the pragmatic listener, I'll skip to the final result.

(54) **Pragmatic listener**:

$$L_1 = \begin{array}{l|ccc} & \mathbf{l} & \mathbf{m} & \mathbf{r} \\ \text{"hat"} & 0.21 & 0.79 & 0 \\ \text{"glasses"} & 0 & 0 & 0 \\ \text{"mustache"} & 0 & 1 & 0 \end{array} \qquad \frac{S_1(m|r)\cdot P(r)}{\Sigma_{m' \in M} S_1(m'|r)\cdot P(r)}$$

a. On hearing "hat" the listener is biased towards middle face (cf. the last game)
b. On hearing "glasses" the listener picks no face
c. On hearing "mustace" the listener picks middle face.

How should we summarize the effect of manipulating salience?

9.7 Incorporating manner

9.7.1 Gricean conciseness

Starting with Kroch 1972, there is a persistant criticism of Gricean theory: the *symmetry problem*.

(55) $U =$ 'Some of the puppies escaped.'
$p =$ *not all the puppies escaped*

a. Contextual premise: Sp knows how many puppies escaped
b. Contextual premise: Kyle is cooperative.
c. By (b), Kyle's utterance will be maximally informative, relevant, and true.
d. The utterance "All the puppies escaped" (= U') is more informative/relevant than U.
e. By (b–d), Sp must lack evidence that U' is true.
f. By (a) and (e), Kyle lacks evidence for U' because it is false.
g. Therefore, p (it is false that all of the puppies escaped)

However, this reasoning *crucially* relies on the choice of U' as "all".

If we reason about why the speaker didn't choose U' as "some but not all", we get the *opposite* inference.

(56) $U =$ 'Some of the puppies escaped.'
$p =$ *all the puppies escaped*

a. Contextual premise: Sp knows how many puppies escaped
b. Contextual premise: Kyle is cooperative.
c. By (b), Kyle's utterance will be maximally informative, relevant, and true.
d. The utterance "Some but not all of the puppies escaped" (= U') is more informative/relevant than U.
e. By (b–d), Sp must lack evidence that U' is true.
f. By (a) and (e), Kyle lacks evidence for U' because it is false.
g. Therefore, p (it is false that some but not all of the puppies escaped, i.e., all of them did)

It seems like Gricean inference can derive the wrong inference. But! My argument, Grice's theory *already* accounts for this problem.

Grice's theory incorporates *manner*, specifically conciseness.

- There's a reason "some"/"all" are preferable to "some but not all": they are shorter.

- So step (e) may not be justified: the speaker had a reason for not choosing "some but not all", i.e., because it's longer than "some".

So manner is crucial to a Gricean theory of implicatures.

9.7.2 Cost in RSA

There's a built-in way to handle manner in RSA: cost.

See Bergen, Levy, and Goodman 2016 for the application of cost to this "some but not all" problem, and Jeong and Collins 2019 for an extension to related problems.

The speaker is the one who chooses messages, so she is the one who cares about cost. To incorporate costs, we first define a function $C : M \mapsto \mathbb{R}$.

(57) $C =$

m	$C(m)$
"some"	0
"all"	0
"some but not all"	4
"some, but please don't get the impression that I mean all, oh dear god no"	15

Let's go back to the original face sizes, but now instead of "mustache", the message is "twirly hairy thing-a-majig".

(58)

(59) a. $[\![\cdot]\!] =$

	l	m	r
"hat"	1	1	0
"glasses"	0	0	0
"t.h.t.mj"	0	1	0

b. for any r, $P(m) = 0.33$ (even priors)

c. $C =$

m	$C(m)$
"hat"	0
"glasses"	0
"t.h.t.mj"	6

The listener agents don't care about cost, so the literal listener is the same as Game 1 (priors are even so they're deleted).

(60) **Literal listener**:

$L_0 =$

	l	m	r
"hat"	0.5	0.5	0
"glasses"	0	0	0
"t.h.t.mj"	0	1	0

$$\frac{[\![m]\!](r)}{\Sigma_{r' \in T}[\![m]\!](r')}$$

The speaker incorporates cost, so we have to update the formula.

(61) **Pragmatic speaker** (with costs): $S(m|r) = \frac{\exp(\ln(L_0(r|m))) - C(m))}{\Sigma_{m' \in M} \exp(\ln(L_0(r|m'))) - C(m'))}$

To calculate the speaker's preferences, we take the listener's preferences and subtract the cost of the message. The 'exp' and 'log' are just to scale it to [0,1].

(62) Step 1: transpose the L_0 table:

	"hat"	"glasses"	"t.h.t.mj"
l	0.5	0	0
m	0.5	0	1
r	0	0	0

$L_0(r|m)$

(63) Step 2: subtract costs and scale:

	"hat"	"glasses"	"t.h.t.mj"
l	0.5	0	0
m	0.5	0	0.0025
r	0	0	0

$\exp(\ln(L_0(r|m))) - C(m))$

(64) Final step: normalize (divide by row total):
Pragmatic speaker (with costs):

$S_1 =$

	"hat"	"glasses"	"t.h.t.mj"
l	1	0	0
m	0.995	0	0.005
r	0	0	0

$\frac{\exp(\ln(L_0(r|m))) - C(m))}{\Sigma_{m' \in M} \exp(\ln(L_0(r|m'))) - C(m'))}$

a. Observing left face, speaker chooses "hat"
b. Observing middle face, speaker is *heavily* biased towards saying "hat" over "twirly hairy thing-a-majig".
c. Observing right face, speaker says nothing.

Calculating the pragmatic listener is just the same as in Game 1 (excluding salience because priors are even), except for the input numbers.

(65) **Pragmatic listener**:

$L_1 =$

	l	**m**	**r**
"hat"	0.501	0.499	0
"glasses"	0	0	0
"t.h.t.mj"	0	1	0

$\frac{S_1(m|r)}{\Sigma_{m' \in M} S_1(m'|r)}$

a. On hearing "hat" the listener is almost evenly split between left and middle face
b. On hearing "glasses" the listener picks no face
c. On hearing "twirly hairy thing-a-majig" the listener picks middle face.

What's the intuitive reason behind result (a)? What would Grice say about this game?

(66) **The key intuition:** cost (here, lengthiness) dampens an expressions effect as an alternative.

Agents reason that the speaker is less likely to use a costly message, even if it's more informative.

Here cost is associated with length, but what else could it be associated with?

a. rudeness? (see Yoon et al 2016)
b. lexical (in)frequency? (e.g., in a corpus)

c. (in)frequency in the immediate interaction? (see Jeong and Collins 2019)
d. social appropriateness? (e.g., slang in a formal setting, formal language in a casual setting)
e. bilingual or bi-dialectal code switching? (maybe SLS theories have insight)
f. other ideas?

9.8 Rationality

A factor which Grice doesn't bring up much: the propensity of an interlocutor to make inferences. What do we expect about:

a. A normal adult?
b. A young child?
c. An adult of low intelligence?
d. An adult with limited ability to pick up on social cues?

One factor built into RSA that we've seen already: the number of iterations in the reasoning process. Speakers and listeners can reason recursively an unlimited amount of times.

(67) from Potts 2013, iterating reasoning 20 times strengthens the implicature. (NB: unlike us, Potts starts with a "literal speaker", but the end result is the same).

	'hat'	'glasses'
r_1	0	1
r_2	0.5	0.5

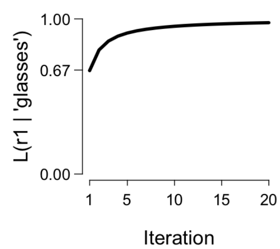
(a) S_0

	r_1	r_2
'hat'	0	1
'glasses'	0.67	0.33

(b) $L(S_0)$

	'hat'	'glasses'
r_1	0	1
r_2	0.75	0.25

(c) $S(L(S_0))$



(d) Highlighted values in tab. (b) up to 20 L/S alternations.

We might hypothesize that children or adults with more limited capabilities might reason to a smaller number of iterations, thus drawing weaker/fewer implicatures.

A second way at getting at this notion is with a value α, a speaker's "rationality parameter". Like cost and salience, this is given at the start of the game.

Let's go back to Game 1 (equal costs and salience), but now the speaker is highly rational, with $\alpha = 4$. Only the speaker cares about α so L_0 is the same (P is deleted due to equal salience).

(68) **Literal listener**:

$L_0 =$

	l	**m**	**r**
"hat"	0.5	0.5	0
"glasses"	0	0	0
"mustache"	0	1	0

$$\frac{[\![m]\!](r)}{\Sigma_{r' \in T}[\![m]\!](r')}$$

The speaker cares about rationality, so we'll update the S_1 formula.

(69) **Pragmatic speaker:** (final definition)

$$S_1(m|r) = \frac{\exp(\alpha \cdot (\ln(L_0(r|m))) - C(m)))}{\Sigma_{m' \in M} \exp(\alpha \cdot (\ln(L_0(r|m'))) - C(m')))}$$

We don't have costs in this game, so we can simplify a bit. Skipping to the final result:

(70) **Pragmatic speaker** (with rationality):

$S_1 =$

	"hat"	"glasses"	"mustache"
l	1	0	0
m	0.059	0	0.941
r	0	0	0

$$\frac{\exp(\alpha \cdot \ln(L_0(r|m))))}{\Sigma_{m' \in M} \exp(\alpha \cdot \ln(L_0(r|m'))))}$$

a. Observing left face, speaker says "hat"
b. Observing middle face, speaker *highly* biased towards "mustache"
c. Observing right face, speaker says nothing.

(71) **Pragmatic listener**:

$L_1 =$

	l	**m**	**r**
"hat"	0.944	0.056	0
"glasses"	0	0	0
"mustache"	0	1	0

$$\frac{S_1(m|r)}{\Sigma_{m' \in M} S_1(m'|r)}$$

a. On hearing "hat" the listener is *heavily* biased towards left face.
b. On hearing "glasses" the listener picks no face
c. On hearing "mustache" the listener picks middle face.

The results are the same as Game 1: for the listener "hat" implies no mustache and for the speaker, "mustache" is a better label for the middle face.

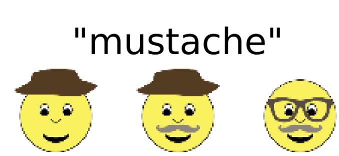
However, the reasoning is much more decisive: the preferences for these results are close to 1 after just one iteration.

Why is α just in the speaker formula? Remember Grice on implicatures: "the speaker thinks (and would expect the hearer to think that the speaker thinks) that it is within the competence of the hearer to work out, or grasp intuitively, that [the implicature is necessary to maintain cooperativity]."

9.9 Higher order implicatures

We can now handle more complex cases like the following:

(72)



Assuming equal costs and salience, and $\alpha = 1$, let's work out both cases on the board.

(73) a. $[\![\cdot]\!] =$ **Game 1**

	l	**m**	**r**
"hat"			
"glasses"			
"mustache"			

b. for any r, $P(r) =$

c. for any m, $C(m) =$

(74) **Literal listener**:

$L_0 =$

	l	**m**	**r**
"hat"			
"glasses"			
"mustache"			

$$\frac{[\![m]\!](r)}{\Sigma_{r' \in T}[\![m]\!](r')}$$

(75) **Pragmatic speaker**:

$S_1 =$

	"hat"	"glasses"	"mustache"
l			
m			
r			

$$\frac{L_0(r|m)}{\Sigma_{m' \in M} L_0(r|m')}$$

(76) **Pragmatic listener**:

$L_1 =$

	l	**m**	**r**
"hat"			
"glasses"			
"mustache"			

$$\frac{S_1(m|r)}{\Sigma_{m' \in M} S_1(m'|r)}$$

(77) a. $[\![\cdot]\!] =$ **Game 2**

	l	**m**	**r**
"hat"			
"glasses"			
"mustache"			

b. for any r, $P(r) =$

c. for any m, $C(m) =$

(78) **Literal listener**:

$L_0 =$

	l	**m**	**r**
"hat"			
"glasses"			
"mustache"			

$$\frac{[\![m]\!](r)}{\Sigma_{r' \in T}[\![m]\!](r')}$$

(79) **Pragmatic speaker**:

$S_1 =$

	"hat"	"glasses"	"mustache"
l			
m			
r			

$$\frac{L_0(r|m)}{\Sigma_{m' \in M} L_0(r|m')}$$

(80) **Pragmatic listener**:

$L_1 =$

	l	**m**	**r**
"hat"			
"glasses"			
"mustache"			

$$\frac{S_1(m|r)}{\Sigma_{m' \in M} S_1(m'|r)}$$

A final fun fact: Frank and Goodman 2012, the paper which proposes this theory, is *one. page. long.*

Bibliography

Bergen, Leon, Roger Levy, and Noah Goodman. 2016. "Pragmatic reasoning through semantic inference". *Semantics and Pragmatics* 9 (20): 1–84.

Burnett, Heather. 2017. "Sociolinguistic interaction and identity construction: The view from game-theoretic pragmatics". *Journal of Sociolinguistics* 21:238–271.

Clark, Herbert H., Robert Schreuder, and Samuel Buttrick. 1983. "Common ground at the understanding of demonstrative reference". *Journal of Verbal Learning and Verbal Behavior* 22 (2): 245–258.

Collins, James N., and Masoud Jasbi. 2013. "A Lewisian Semantics for the English Definite Determiner". Ms., Stanford University.

Degen, Judith, Michael Henry Tessler, and Noah D. Goodman. 2015. "Wonky worlds: Listeners revise world knowledge when utterances are odd". In *Proceedings of the 37th Annual Meeting of the Cognitive Science Society*.

Frank, Michael C., and Noah D. Goodman. 2012. "Predicting pragmatic reasoning in language games". *Science* 336 (6084): 998.

— . 2014. "Inferring word meanings by assuming that speakers are informative". *Cognitive Psychology* 75:80–96.

Hirschberg, Julia. 1985. "A theory of scalar implicature". PhD thesis, University of Pennsylvania.

Horn, Laurence R. 1972. "On the semantic properties of logical operators in English". PhD thesis, UCLA.

Jeong, Sunwoo, and James N. Collins. 2019. "Updating alternatives in pragmatic competition". Paper presented at the 20th Annual SemFest, Stanford, CA.

Kao, Justine T., Leon Bergen, and Noah D. Goodman. 2014. "Formalizing the pragmatics of metaphor understanding". In *Proceedings of the 36th Annual Meeting of the Cognitive Science Society*, 719–724. Wheat Ridge, CO: Cognitive Science Society.

Kroch, Anthony. 1972. “Lexical and inferred meanings for some time adverbials”. In *Quarterly Progress Reports of the Research Laboratory of Electronics* 104. Cambridge, MA: MIT Press.

Lassiter, Daniel, and Noah D. Goodman. 2017. “Adjectival vagueness in a Bayesian model of interpretation”. *Synthese* 194 (10): 3801–3836.

Potts, Christopher. 2013. “Conversational implicature: Interacting with grammar”. Paper presented at the Michigan Philosophy–Linguistics Workshop.

Potts, Christopher, et al. 2016. “Embedded implicatures as pragmatic inferences under compositional lexical uncertainty”. *Journal of Semantics* 33 (4): 755–802.

Qing, Ciyang, and Reuben Cohn-Gordon. 2019. “Non-descriptive/use-conditional meaning in Rational Speech-Act models”. To appear in *the Proceedings of Sinn und Bedeutung 23*.

Rosenberg, Seymour, and Bertram D. Cohen. 1964. “Speakers’ and listeners’ processes in a word communication task”. *Science* 145:1201–1203.

Scontras, Gregory, Michael Henry Tessler, and Michael Franke. 2018. “Probabilistic language understanding: An introduction to the Rational Speech Act framework”. Retrieved 2019-4-5 from https://www.problang.org.

Stiller, Alex, Noah D. Goodman, and Michael C. Frank. 2011. “Ad-hoc scalar implicature in adults and children”. In *Proceedings of the 33rd Annual Meeting of the Cognitive Science Society*.

Tessler, Michael Henry, and Michael Franke. 2018. “Not unreasonable: Carving vague dimensions with contraries and contradictions”. In *Proceedings of the 40th Annual Meeting of the Cognitive Science Society*.

Yoon, Erica J., et al. 2017. ““I won’t lie, it wasn’t amazing”: Modeling polite indirect speech”. In *Proceedings of the 39th Annual Meeting of the Cognitive Science Society*.